AS & A Level Mathematics (9709) Paper 5

[Probability & Statistics 1]

Exam Series: May 2015 – May 2022

Format Type A:
Answers to all questions are provided as an appendix

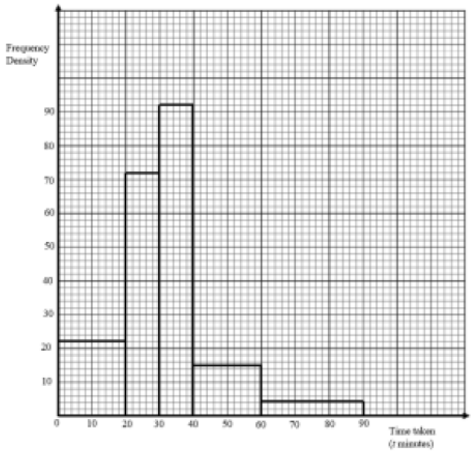
Appendix A

Answers

1. 9709_m22_ms_52 Q: 3

<table>
<tr><th>Question</th><th>Answer</th><th>Marks</th><th>Guidance</th></tr>
<tr><td rowspan="4">(a)</td><td rowspan="3"><table><tr><td>Class Width</td><td>30</td><td>15</td><td>20</td><td>10</td><td>25</td></tr><tr><td>Frequency Density</td><td>0.7</td><td>2</td><td>3.4</td><td>8.6</td><td>1.8</td></tr></table></td><td>M1</td><td>At least 4 frequency densities calculated</td></tr>
<tr><td>A1</td><td>All heights correct on graph</td></tr>
<tr><td>B1</td><td>Bar ends at 0·5, 30·5, 45·5, 65.5, 75.5, 100.5 (at axis), 5 bars drawn, condone 0 in first bar
0.5 ⩽ time axis ⩽ 100.5, linear scale with at least 3 values indicated.</td></tr>
<tr><td>B1</td><td>Axes labelled: Frequency density (fd), time (t) and mins (or appropriate title). Linear fd scale, with at least 3 values indicated 0 ⩽ fd axis ⩽ 8·6</td></tr>
<tr><td></td><td></td><td>4</td><td></td></tr>
<tr><td rowspan="2">(b)</td><td>66 – 75</td><td>B1</td><td>Condone 65.5 – 75.5</td></tr>
<tr><td></td><td>1</td><td></td></tr>
<tr><td rowspan="2">(c)</td><td>Distribution is not symmetrical</td><td>B1</td><td>Or skewed, ignore nature of skew</td></tr>
<tr><td></td><td>1</td><td></td></tr>
</table>

2. 9709_s22_ms_51 Q: 3

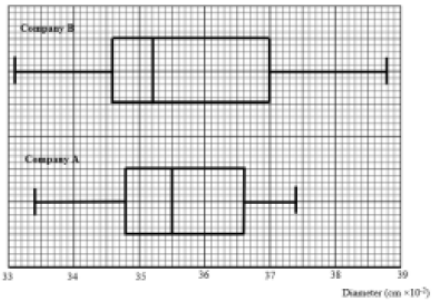
Question	Answer	Marks	Guidance
(a)	<table><tr><td>Class width</td><td>20</td><td>10</td><td>10</td><td>20</td><td>30</td></tr><tr><td>Frequency density</td><td>22</td><td>72</td><td>92</td><td>15</td><td>4</td></tr></table> 	M1	At least 4 frequency densities calculated (Frequency ÷ class width, e.g. $\frac{440}{20}\left(condone\ \frac{440}{19.5}, \frac{440}{20.5}\right)$ Accept unsimplified, may be read from graph using *their* scale
		A1	All heights correct on graph **NOT FT**
		B1	Bar ends at [0,] 20, 30, 40, 60, 90 at axis with a horizontal linear scale with at least 3 values indicated. 0 ⩽ horizontal scale ⩽ 90
		B1	Axes labelled frequency density (fd), time (t) and minutes (mins) or in a title. Linear vertical scale, with at least 3 values indicated 0 ⩽ vertical axes ⩽ 92 (condone 90 used).
		4	

Question	Answer	Marks	Guidance
(b)	<table><tr><td>Midpoints</td><td>10</td><td>25</td><td>35</td><td>50</td><td>75</td></tr></table>	B1	At least 4 correct midpoints seen
	[Mean = 31.44 given] [Variance = $\frac{440\times10^2+720\times25^2+920\times35^2+300\times50^2+120\times75^2}{2500}-31.44^2$] $=\frac{44000+450000+1127000+750000+675000}{2500}-31.44^2$ [$=\frac{3046000}{2500}-31.44^2=229.9264$] Or Variance = $\frac{440(10-31.44)^2+720(25-31.44)^2+920(35-31.44)^2+300(50-31.44)^2+120(75-31.44)^2}{2500}$ $=\frac{202256+29860+11659+103342+227697}{2500}=\frac{574814}{2500}=229.9264$	M1	Correct formula for variance or standard deviation (− mean^2 included with *their* midpoints (not upper bound, lower bound, class width, frequency density, frequency or cumulative frequency) and *their* $\sum f$ if calculated. Condone 1 data error.
	Standard deviation = 15.2	A1	WWW, allow 15.16[3…]
		3	
(c)	30–40	B1	
		1	
(d)	Stays the same, data still in same intervals	B1	Frequencies unchanged
		1	

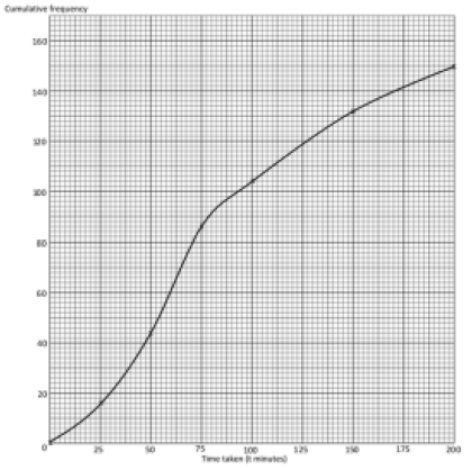
3. 9709_s22_ms_52 Q: 1

Question	Answer	Marks	Guidance
	$\Sigma x - \Sigma 200 = \Sigma(x - 200)$	B1	Forming a correct 3-term (linear) equation from Σx, $\Sigma 200$ and $\Sigma(x-200)$. Accept $6846 - 200n = 446$ OE. Condone 1 sign error.
	$\Sigma 200 = 200n$	B1	SOI
	$[200n = 6846 - 446 = 6400]$ $n = 32$	B1	WWW
		3	

4. 9709_s22_ms_52 Q: 3

Question	Answer	Marks	Guidance
(a)	Median = 0.355	B1	Identified condone Q2.
	[IQR =] 0.366 – 0.348	M1	$0.365 \leqslant$ UQ $\leqslant 0.369 - 0.343 \leqslant$ LQ $\leqslant 0.349$. Subtraction may be implied by answer.
	0.018	A1	If 0/3 scored SC B1 for figs Median = 355 IQR = 18.
		3	
(b)	Box-and-whisker plot on provided grid  	B1	All 5 key values for *B* plotted accurately in standard format using *their* scale. Labelled *B*. Check accuracy in the middle of vertical line.
		B1 FT	All 5 key values for *A*, FT from part **3(a)**, plotted in standard format accurately using *their* scale. Labelled *A*. Check accuracy in the middle of vertical line.
		B1	Whiskers not through box for both, not drawn at corners of boxes, single linear scale with at least 3 values stated, covering at least 0.34 to 0.38 and labelled diameter (*d* etc) and cm. Accept as a title.
		3	If both plots attempted and plot(s) not labelled, SC B1 for at least 1 fully correct set of values plotted.
(c)	A comparison in context	B1	Single comment comparing spread or central tendency in context. Must reference either diameter or pipes. Not a simple numerical comparison of statistical values such as median, range, IQR or min/max.
		1	

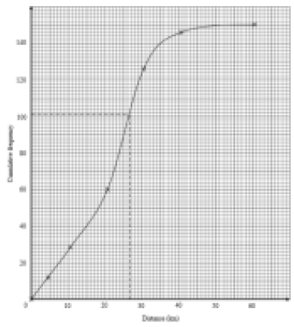
5. 9709_s22_ms_53 Q: 1

Question	Answer	Marks	Guidance
(a)	Cumulative frequency (cf) graph	M1	At least 3 points plotted accurately at class upper end points (25,16) (50,44) (75,86) (100,104) (150, 132) (200, 150). Linear cf scale $0 \leqslant cf \leqslant 150$ and linear time scale $0 \leqslant$ time(mins) $\leqslant 200$ with at least 3 values identified on each axis.
		A1	All points plotted correctly, curve drawn (within tolerance) and joined to (0,0). Axes labelled cumulative frequency (cf), time (t) and minutes (min), or a suitable title.
		2	
(b)	Line from cumulative frequency = 30 to meet graph at t is between 37.5 and 42	B1 FT	Not from wrong working. Must be an increasing cumulative frequency graph.
		1	

6. 9709_s22_ms_53 Q: 2

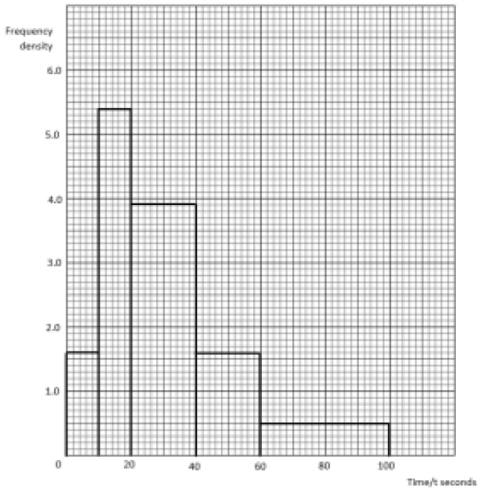
Question	Answer	Marks	Guidance
(a)	$\left[\frac{123.4}{20} = \right] 6.17$	B1	Accept 6 m 17 cm, $\frac{1234}{200}$.
		1	
(b)	$\frac{10\text{th}+11\text{th}}{2} = \frac{5.4+5.5}{2} = 5.45$ (m)	B1	Accept 5 m 45 cm.
		1	
(c)	The mean is unduly influenced by an extreme value, 19.4.	B1	Comment must be within context.
		1	

7. 9709_m21_ms_52 Q: 5

Question	Answer	Marks	Guidance
(a)	<table><tr><td>Distance</td><td>0-4</td><td>5-10</td><td>11-20</td><td>21-30</td><td>31-40</td><td>41-60</td></tr><tr><td>Upper boundary</td><td>4·5</td><td>10·5</td><td>20·5</td><td>30·5</td><td>40·5</td><td>60·5</td></tr><tr><td>Cumulative frequency</td><td>12</td><td>28</td><td>60</td><td>126</td><td>146</td><td>150</td></tr></table>	**B1**	Correct cumulative frequencies seen (may be by table or plotted accurately on graph), condone 12 not stated.
		**B1**	Axes labelled 'distance (or d) [in] km' from 0 to 60 **and** 'cumulative frequency' (or cf) from 0 to 150.
		M1	At least 5 points plotted at upper end points for *d* (allow upper boundary ±0·5) with a linear scale for distance, condone 0 – 4 interval inaccurate, no scale break on axis. Not bar graph/histogram unless clear indication of upper end point only of each bar.
		A1	All plotted correctly at correct upper end points (4.5 etc.) with both scales linear ($0 \leqslant d \leqslant 60$, $0 \leqslant$ cf $\leqslant 150$), curve drawn accurately joined to (0,0), cf line>150, no daylight if >150.
		4	
(b)	70% of 150 = 105	**M1**	105 seen or implied by indication on grid.
	Approx. 27	**A1 FT**	Strict FT *their* increasing cumulative frequency graph, use of graph must be seen. If no clear evidence of use of graph: **SC B1 FT** correct value from *their* increasing cumulative frequency graph.
		2	

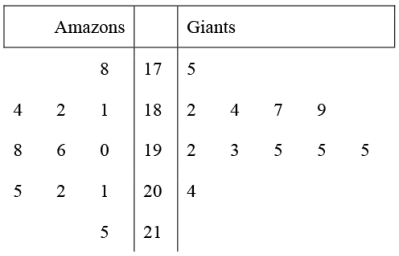
Question	Answer	Marks	Guidance
(c)	Midpoints: 2.25, 7.5, 15.5, 25.5, 35.5, 50.5	**B1**	At least 5 correct midpoints seen.
	$\text{Mean} = \frac{2.25\times12+7.5\times16+15.5\times32+25.5\times66+35.5\times20+50.5\times4}{150}$ $= \frac{27+120+496+1683+710+202}{150}$	**M1**	Using 6 midpoint attempts (e.g. 2·25 ±0·5), condone one error not omission, multiplied by frequency, accept unevaluated, denominator either correct or *their* Σ frequencies.
	$\left[=\frac{3238}{150}\right] = 21.6,\ 21\frac{44}{75}$	**A1**	Evaluated, WWW, accept 21·5[866…].
		3	

8. 9709_s21_ms_51 Q: 5

Question	Answer	Marks	Guidance
(a)	<table><tr><td>Class width</td><td>10</td><td>10</td><td>20</td><td>20</td><td>40</td></tr><tr><td>Frequency Density</td><td>1.6</td><td>5.4</td><td>3.9</td><td>1.6</td><td>0.5</td></tr></table> 	M1	At least 4 frequency densities calculated, accept unsimplified. May be read from graph using *their* scale, 3SF or correct
		A1	All heights correct on graph
		B1	Bar ends at 0, 10, 20 …, etc. with a horizontal linear scale with at least 3 values indicated, 0 ⩽ horizontal axis ⩽ 100
		B1	Axes labelled: Frequency density (fd), time (t) and seconds. Linear vertical scale, with at least 3 values indicated 0 ⩽ vertical axis ⩽ 5.4
		4	

Question	Answer	Marks	Guidance
(b)	$\text{Mean} = \left[\frac{16\times5+54\times15+78\times30+32\times50+20\times80}{200}\right]$ $= \frac{80+810+2340+1600+1600}{200}$	M1	Uses at least 4 midpoint attempts (e.g. 5 ± 0.5). Accept unsimplified expression, denominator either correct or *their* Σfrequencies
	$\left[\frac{6430}{200} =\right] 32\frac{3}{20}$ or 32.15	A1	Accept 32.2
		2	
(c)	A value in correct UQ (40–60) – a value in correct LQ (10–20)	M1	
	Greatest possible value is 60 – 10 = 50	A1	Condone $49.\dot{9}$
		2	

9. 9709_s21_ms_52 Q: 7

Question	Answer	Marks	Guidance
(a)	Includes all data	B1	Reference to *either* including all/raw data or further statistical processes are possible that cannot be found using data from box-and-whisker, eg frequency, mean, mode or standard deviation **not** only median, IQR, range or spread which can be found from both.
		1	
(b)	 Key: 1|18|2 means 181 cm for Amazons and 182 cm for Giants	B1	Correct stem can be upside down, ignore extra values
		B1	Correct Amazons labelled on left, leaves in order from right to left and lined up vertically (less than halfway to next column), no commas or other punctuation.
		B1	Correct Giants labelled on same diagram, leaves in order and lined up vertically (less than halfway to next column), no commas or other punctuation.
		B1	Correct single key for their diagram, need both teams identified and 'cm' stated at least once here or in leaf headings or title. **SC** for if 2 separate diagrams drawn, award **SCB1** if both keys meet these criteria (Max B1, B0, B0, B1)
		4	
(c)	[UQ = 202 (cm), LQ = 182 (cm)] [IQR =] 202 – 182 = 20 (cm)	M1	$201 \leqslant UQ \leqslant 205 - 181 \leqslant LQ \leqslant 184$
		A1	WWW
		2	

Question	Answer	Marks	Guidance
(d)	$[\Sigma_{11} = 2132$ $\Sigma_{15} = 191.2 \times 15 = 2868\ ]$	B1	Both Σ_{11} and Σ_{15} found. Accept unevaluated.
	their 2868 = *their* 2132 + (180 + 185 + 190) + h	M1	Forming an equation for the height using *their* Σ_{11} and Σ_{15}.
	181 (cm)	A1	
	Alternative method for Question 7(d)		
	$[\Sigma_{15} = 191.2 \times 15 = 2868$ $\Sigma_{15} = 2687 + h\ ]$	B1	Σ_{15} found using the mean and raw data methods. Accept unevaluated.
	their 2868 = *their* 2687 + h	M1	Forming an equation for the height using *their* Σ_{15} expressions.
	181 (cm)	A1	
	Alternative method for Question 7(d)		
	$[\Sigma_{15} = 2687 + h$ $\frac{\Sigma_{15}}{15} = 191.2\ ]$	B1	Σ_{15} found using raw data method and statement on calculating new mean. Accept unevaluated.
	$\frac{\text{their } 2687 + h}{15} = 191.2$	M1	Forming an equation for the height using *their* Σ_{15} expressions
	181 (cm)	A1	
		3	**N.B.** All methods can be presented as a logical numerical argument which can be condoned if clear.

10. 9709_s21_ms_53 Q: 1

Question	Answer	Marks	Guidance
(a)	60	B1	Accept 60 or 61. No decimals
		1	
(b)	65% of 160 = 104	M1	0.65 × 160 (=104) seen unsimplified or implied by use on graph
	136 (cm)	A1	Use of graph must be seen. **SCB1** correct value (136 only) if neither 104 nor use of graph are evident
		2	
(c)	UQ: 150 LQ: 76 IQR = 150 – 76 = 74 [cm]	M1	UQ – LQ ; 148 ⩽ UQ ⩽ 152; 74 ⩽ LQ ⩽ 78.
		A1	Must be from 150 - 76
		2	

11. 9709_s21_ms_53 Q: 3

Question	Answer	Marks	Guidance
(a)	Mean height $= \frac{\Sigma x + \Sigma y}{6+11} = \frac{1050+1991}{6+11} = \frac{3041}{17}$	M1	Use of appropriate formula with values substituted, accept unsimplified.
	178.9	A1	Allow 178.88, $178\frac{15}{17}$, 179
		2	

Question	Answer	Marks	Guidance
(b)	$\frac{\Sigma x^2 + \Sigma y^2}{6+11} = \frac{193700+366400}{6+11}$	M1	Use of appropriate formula with values substituted, accept unsimplified.
	$\text{Sd}^2 = \frac{560100}{17} - \textit{their}\,178.88^2\,[=948.289]$	M1	Appropriate variance formula using *their* mean^2, accept unsimplified expression.
	Standard deviation = 30.8	A1	Accept 30.7
		3	

12. 9709_w21_ms_51 Q: 2

Question	Answer	Marks	Guidance
(a)	$\left[\frac{\Sigma x}{40} - k = \frac{\Sigma(x-k)}{40}\right]$ $\frac{40\times 34}{40} - k = \frac{520}{40}$	M1	Forms an equation involving Σx, $\Sigma(x\text{-}k)$ and k. Accept at a numeric stage with k.
	$k\,[=34-13]=21$	A1	Evaluated.
		2	

Question	Answer	Marks	Guidance
(b)	$\text{Var} = \left[\frac{\Sigma(x-k)^2}{40} - \left(\frac{\Sigma(x-k)}{40}\right)^2\right] = \frac{9640}{40} - \left(\frac{520}{40}\right)^2 = [241 - 13^2 =]$	M1	Values substituted into an appropriate variance formula, accept unsimplified.
	72	A1	
		2	

13. 9709_w21_ms_51 Q: 6

Question	Answer	Marks	Guidance
(a)	Rebels | Sharks 6 | 6 8 9 8 5 | 7 | 1 2 4 5 5 6 8 9 6 5 4 3 2 2 0 | 8 | 3 3 4 5 6 9 5 3 | 9 | 2 2 | 10 Key: 8 | 7 | 2 means 78 kg for Rebels and 72 kg for Sharks	**B1**	Correct stem, ignore extra values (not in reverse).
		B1	Correct Rebels labelled on left, leaves in order from right to left and lined up vertically, no commas.
		B1	Correct Sharks labelled on same diagram, leaves in order and lined up vertically, no commas.
		B1	Correct key for their diagram, need both teams identified and 'kg' stated at least once here or in leaf headings or title. **SC** If 2 separate diagrams drawn, **SC B1** if both keys meet these criteria.
		4	

Question	Answer	Marks	Guidance
(b)	Median = 84 (kg)	**B1**	
	[UQ = 93, LQ = 80] 93 – 80	**M1**	95 ⩽ UQ ⩽ 89 – 79 ⩽ LQ ⩽ 82
	[IQR =] 13 (kg)	**A1**	WWW
		3	
(c)	Box and whisker with end points 75 and 102	**B1**	Whiskers drawn to correct end points not through box, not joining at top or bottom of box.
	Median and quartiles plotted as found in **(b)**	**B1 FT**	Quartiles and median plotted as box graph.
		2	
(d)	e.g. Average weight of Rebels is higher than average weight of Sharks	**B1**	Acceptable answers refer to: Range, skew, central tendency within context. E.g. range of Rebels is greater **B0**. Range of weights of the rebels is greater **B1**. Simple value comparison insufficient.
		1	

14. 9709_w21_ms_52 Q: 7

Question	Answer	Marks	Guidance
(a)	Cumulative frequency graph drawn	**B1**	Axes labelled 'cumulative frequency' (or cf) from 0 to at least 140 and 'distance (or d) [in] m' from 0 to at least 1600, linear scales with at least 3 values stated.
		B1	All plotted correctly at correct upper end points (200 etc.) **curve** drawn accurately joined to (0, 0) (straight line segments B0) but no daylight above 140. Cf scale no less than 2 cm = 20 children .
		2	

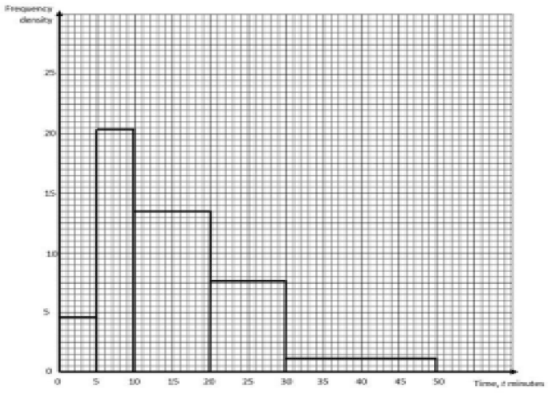
Question	Answer	Marks	Guidance
(b)	[UQ at 75% of 140 = 105, LQ at 25% of 140 = 35] [IQR:] 700 – 260	**M1**	Accept 660 ⩽ UQ ⩽ 720 – 240 ⩽ LQ ⩽ 290. If values are outside our range, FT providing scales linear and increasing cf drawn.
	440	**A1**	Accept correct evaluation of 660 ⩽ *their* UQ ⩽ 720 – 240 ⩽ *their* LQ ⩽ 290 with clear indication that graph has been used for at least one of 105 or 35.
		2	

Question	Answer	Marks	Guidance
(c)	[Mean =] $\frac{16\times100+30\times250+42\times400+34\times700+12\times1050+6\times1400}{140}$	B1	Frequencies: 16, 30, 42, 34, 12, 6 Mid-points: 100, 250, 400, 700, 1050, 1400 5 or 6 correct frequency values seen.
		B1	5 or 6 correct midpoint values seen.
		M1	Values substituted into mean formula using *their* midpoints which must be in the class – condone 1 data error. Accept $\frac{1600+7500+16\,800+23\,800+12\,600+8400}{140}$ or $\frac{70\,700}{140}$. Condone $\frac{70\,770}{140}$ for **M1**.
	505	A1	WWW
	Variance = $\frac{16\times100^2+30\times250^2+42\times400^2+34\times700^2+12\times1050^2+6\times1400^2}{140}$ -505^2	M1	Values substituted into variance formula using (*their* mean)^2 and *their* midpoints and *their* frequencies (including for denominator). Accept unsimplified. Condone 1 data error. Accept: $[\frac{160\,000+1\,875\,000+6\,720\,000+16\,660\,000+13\,230\,000+11\,760\,000}{140}$ or $\frac{50\,405\,000}{140}$ or 360 035.7143] $-[505^2$ or 255 025] If formula stated accept 105 010 or 105 011 WWW.
	S.d. = $[\sqrt{105\,010.7}=]$ 324	A1	WWW
		6	

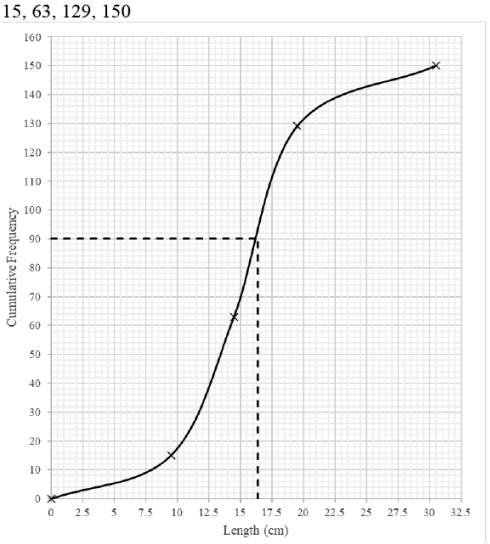
15. 9709_w21_ms_53 Q: 2

Question	Answer	Marks	Guidance
(a)	Lakeview | Riverside 9 4 0 | 1 | 8 8 8 7 6 2 | 2 | 0 1 3 4 5 5 3 2 0 | 3 | 0 6 7 1 | 4 Key: 6|2|3 means 26m for Lakeview and 23m for Riverside	B1	Correct stem, ignore extra values.
		B1	Correct Lakeview labelled on left, leaves in order from right to left and lined up vertically, no commas.
		B1	Correct Riverside labelled on same diagram, leaves in order and lined up vertically, no commas.
		B1	Correct key for their diagram, need both teams identified and 'm' stated at least once here or in leaf headings or title. **SC** If 2 separate diagrams drawn: **SC B1** if both keys meet these criteria.
		4	
(b)	UQ = 32, LQ = 19	M1	(30 ⩽ UQ ⩽ 33) – (14 ⩽ LQ ⩽ 22)
	IQR = 32 – 19 = 13	A1	WWW
		2	

16. 9709_w21_ms_53 Q: 3

Question	Answer	Marks	Guidance
(a)	Cw: 5 5 10 10 20	**M1**	At least 4 frequency densities calculated (f/cw), accept unsimplified and class widths ±1 of true values. May be implied by graph.
	Fd: 4.6 20.4 13.5 7.6 1.2	**A1**	All heights correct on graph **NOT FT**
		**B1**	Bar ends at 0, 5, 10, 20, 30, 50 clear intention not to draw at 4.5 or 5.5 etc.
		B1	Axes labelled: Frequency density (fd), time (t) and mins (or appropriate title). Linear scales between 0 and 20.4 or above on vertical axis, and 0 and 50 or above on the horizontal axis. (Axes may be reversed.)
		4	
(b)	$\frac{2.5\times23+7.5\times102+15\times135+25\times76+40\times24}{360}$	**M1**	Uses at least 4 midpoint attempts (e.g. 2.5 ± 0.5) in correct formula, accept unsimplified expression, denominator either correct or *their* Σfrequencies .
	$\left[\frac{5707.5}{360}=\right]15.9,\ 15\frac{41}{48}$	**A1**	Evaluated.
		2	

17. 9709_m20_ms_52 Q: 7

Question	Answer	Marks	Guidance
(a)	15, 63, 129, 150	B1	Correct cumulative frequencies seen (may be on graph)
		B1	0 ⩽ Horizontal axis ⩽ 30, 0 ⩽ vertical axis ⩽ 150 Labels correct: length cm, cf
		M1	At least 3 points plotted at upper end points (e.g. allow 9, 9.5, 10) with a linear horizontal scale.
		A1	Linear vertical scale, all points at correct upper end points (9.5 etc.), curve drawn accurately, joined to (0,0) (condone (–0.5, 0))
		4	
(b)	60% of 150 = 90	M1	90 seen or implied by use on graph
	Approx. 16.5 [cm]	A1FT	FT *their* increasing cumulative frequency graph, Use of graph must be seen. If no clear evidence of use of graph **SCB1FT** correct value from *their* graph
		2	

Question	Answer	Marks	Guidance
(c)	Midpoints: 4.75, 12, 17, 25	M1	At least 3 correct midpoints used (39449.4375 implies M1)
	Var $= \dfrac{4.75^2 \times 15 + 12^2 \times 48 + 17^2 \times 66 + 25^2 \times 21}{150} - 15.295^2$	M1	Using midpoints ±0.5 in correct var formula, including subtraction of *their* μ^2.
	= 29.1	A1	
		3	

18. 9709_s20_ms_51 Q: 7

Question	Answer	Marks
(a)	Class widths: 10, 5, 15, 20, 10	M1
	Frequency density = frequency/*their* class width: 1.8, 4.8, 2, 1, 0.8	M1
	All heights correct on diagram (using a linear scale)	A1
	Correct bar ends	B1
	Bar ends: 10.5, 15.5, 30.5, 50.5, 60.5	B1
		5
(b)	11 – 15 and 31 – 50	B1
	Greatest IQR = 50 – 11 = 39	B1
		2
(c)	$\text{Mean} = \frac{18\times5.5+24\times13+30\times23+20\times40.5+8\times55.5}{100} = \frac{2355}{100} = 23.6$	B1
	$\text{Var} = \frac{18\times5.5^2+24\times13^2+30\times23^2+20\times40.5^2+8\times55.5^2}{100} - \text{mean}^2$	M1
	$\frac{77917.5}{100} - \text{mean}^2 = 224.57$	A1
	Standard deviation = 15.0 (**FT** *their* variance)	A1 FT
		4

19. 9709_s20_ms_52 Q: 1

Question	Answer	Marks
	$\Sigma x - 50n = 144$	B1
	$50n + 144 = 944$	M1
	$n = 16$	A1
		3

20. 9709_s20_ms_52 Q: 3

Question	*Answer*	Marks
(a)	Median = 0.238	B1
	UQ = 0.245, LQ = 0.231, So IQR = 0.245 – 0.231	M1
	0.014	A1
		3

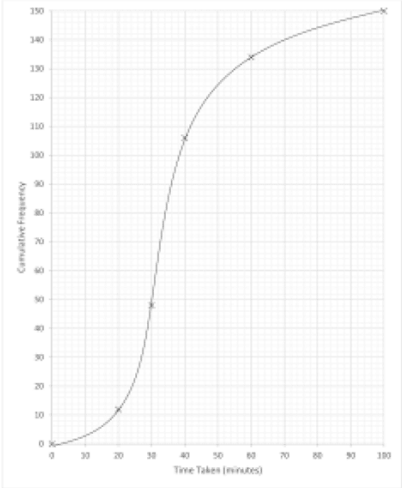
Question	Answer	Marks
(b)	<table><tr><td></td><td></td><td>LQ</td><td>M</td><td>UQ</td><td></td></tr><tr><td>A</td><td>0.220</td><td>0.231 **FT**</td><td>0.238 **FT**</td><td>0.245 **FT**</td><td>0.254</td></tr><tr><td>B</td><td>0.211</td><td>0.224</td><td>0.232</td><td>0.243</td><td>0.256</td></tr></table>	
	Medians and quartiles correctly plotted for *A* or *B*	**B1**
	End points correct for *A* or *B*	**B1**
	Completely correct, including scale	**B1**
		3
(c)	Lengths of rods produced by machine *A* are longer. (**B1** for comparison of central tendency)	**B1**
	Lengths of rods produced by machine *A* are less spread out (**B1** for comparison of spread)	**B1**
		2

21. 9709_s20_ms_53 Q: 6

Question	Answer	Marks
(a)	<table><tr><td>A</td><td></td><td>B</td></tr><tr><td></td><td>2</td><td>6</td></tr><tr><td>5 2 0</td><td>3</td><td>0 1 5 8</td></tr><tr><td>9 7 2 1 1</td><td>4</td><td>1 2 2 7 9</td></tr><tr><td>3 2</td><td>5</td><td>2</td></tr><tr><td>4</td><td>6</td><td></td></tr></table> KEY 1 | 4 | 2 means \$41 000 for A and \$42 000 for B	
	Correct stem	**B1**
	Correct A on LHS	**B1**
	Correct B on same diagram	**B1**
	Correct key for *their* diagram, both companies identified and correct units	**B1**
		4
(b)	Median = [\$]42 000	**B1**
	LQ = [\$]35 000 UQ = [\$]52 000	**B1**
	IQR = [\$]17 000 (**FT** if $49000 \leqslant UQ \leqslant 53000 - 32000 \leqslant LQ \leqslant 41000$)	**B1 FT**
		3

Question	Answer	Marks
(c)	Sum of given 11 numbers is 433 000	**M1**
	Sum of 12 numbers, including new = $38\,500 \times 12 = 462\,000$	**M1**
	Difference = new salary = [\$]29 000	**A1**
		3

22. 9709_w20_ms_51 Q: 6

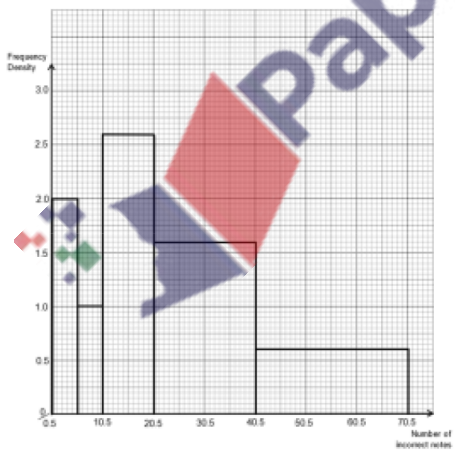
Question	Answer	Marks	Guidance
(a)		M1	At least 4 points plotted at upper end points, with both scales linear with at least 3 values indicated
	Correct cumulative frequency curve	A1	All plotted correctly with curve drawn joined to (0, 0), axes labelled cumulative frequency, time, minutes
		2	
(b)	$150 \times 0{\cdot}76 = 114$	M1	114 SOI, may be on graph
	$k = 45$ (mins)	A1 FT	Clear indication that *their* graph has been used, tolerance ±1mm
		2	

Question	Answer	Marks	Guidance
(c)	Frequencies: 12 36 58 28 16	B1	Correct frequencies seen
	Mean $= \dfrac{10\times12+25\times36+35\times58+50\times28+80\times16}{150}$	B1	At least 4 correct midpoints seen and used
	$\dfrac{120+900+2030+1400+1280}{150}$	M1	Correct formula with *their* midpoints (not upper boundary, lower boundary, class width or frequency density).
	$38.2, 38\dfrac{1}{5}$	A1	
	Variance $= \dfrac{12\times10^2+36\times25^2+58\times35^2+28\times50^2+16\times80^2}{150} - mean^2$ $= \dfrac{1200+22500+71050+70000+102400}{150} - mean^2$	M1	Substitute *their* midpoints and frequencies (condone use of cumulative frequency) in correct variance formula, must have '– *their* mean^2'
	(Standard deviation $= \sqrt{321.76}$) $= 17.9$	A1	
		6	

23. 9709_w20_ms_52 Q: 5

Question	Answer	Marks	Guidance
(a)	*Dados* | | *Linva* 8 6 | 0 | 0 2 9 6 5 2 0 0 | 1 | 0 1 2 5 6 8 2 | 2 | 6 | 3 | 2 6 2 | 4 | 0 KEY 6| 3| 2 means 36 cm (snow) in Dados and 32 cm (snow) in Linva	**B1**	Correct stem can be upside down, ignore extra values
		B1	Correct Dados labelled, leaves in order and lined up vertically (less than midway to next column), no commas etc, no extra terms
		B1	Correct Linva on opposite side of stem labelled, leaves in order and lined up vertically (less than midway to next column), no commas etc, no extra terms
		B1	Correct single key for their diagram, need both resorts identified and 'cm' stated at least once here or in leaf headings or title. **SC** If 2 separate diagrams drawn, **SCB1** if both keys meet these criteria B0B1B0SCB1 max.
		4	
(b)	Median or Q2 = 15 (cm)	**B1**	Correct
	UQ or Q3 = 28 cm, LQ or Q1 = 10 cm IQR = 28 – 10	**M1**	$22 \leqslant UQ \leqslant 36 - 8 \leqslant LQ \leqslant 10$
	18 (cm)	**A1**	WWW
		3	
(c)	On average the snowfall in Davos is higher	**B1 FT**	FT from *their* **5(b)** values for Dados. Statement comparing central tendency in context
	The amount of snowfall in Linva varies more than in Davos	**B1 FT**	Statement comparing spread in context Note: simply stating and comparing the values is not sufficient.
		2	

24. 9709_w20_ms_53 Q: 7

Question	Answer	Marks	Guidance
(a)	Class widths: 5, 5, 10, 20, 30 Frequency density: 2, 1, 2.6, 1.6, 0.6	**M1**	At least 3 class widths correct and used in a calculation
		M1	At least 3 correct frequency densities unsimplified – FT *their* class widths
		**A1**	All correct heights on a histogram using a linear vertical scale from zero – no FT
		B1	Correct upper bar ends (5.5, 10.5, 20.5, 40.5, 70.5) and 4 correct lower bar ends of 5.5, 10.5, 20.5, 40.5. Condone 0 or 1.
		B1	Linear scales with at least 3 values indicated on each axis, vertical scale from 0, axes labelled 'fd' and 'no. of (incorrect) notes', or better.
		5	
(b)	LQ: 11 – 20 UQ: 21 – 40	**B1**	Both UQ and LQ correct
	Greatest IQR = 40 – 11 = 29	**B1 FT**	Subtract lower end of *their* LQ interval from upper end of *their* UQ interval
		2	

Question	Answer	Marks	Guidance
(c)	Midpoints: 3 8 15.5 30.5 55.5	M1	At least 4 midpoints correct and used
	Mean $= \frac{3\times10+8\times5+15.5\times26+30.5\times32+55.5\times18}{91}$ $= \frac{30+40+403+976+999}{91}$ $= \frac{2448}{91}$	M1	Correct formula with *their* midpoints (not upper boundary, lower boundary, class width, frequency density, frequency or cumulative frequency)
	26.9, $26\frac{82}{91}$	A1	Accept 26 or 27
		3	

25. 9709_m19_ms_62 Q: 2

Question	Answer	Marks	Guidance
(i)	$\sigma^2 = \frac{\Sigma(x-c)^2}{n} - \left(\frac{\Sigma(x-c)}{n}\right)^2$ $3.2^2 = \frac{3099.2}{40} - \left(\frac{\Sigma(x-c)}{40}\right)^2$	M1	Use correct formula with values substituted
	$\left(\frac{\Sigma(x-c)}{40}\right)^2 = 67.24$: $\Sigma(x-c) = 40\times\sqrt{67.24}$	M1	Rearrange to make *their* $\left(\frac{\Sigma(x-c)}{40}\right)^2$ the subject, unsimplified.
	= 328	A1	Exact value, cao
		3	
(ii)	$\Sigma x - 40c =$ *their* **(i)** Mean $= \frac{\textit{their (i)}}{40} + 50$ = 58.2	B1FT	FT *their* **(i)**
		1	

26. 9709_m19_ms_62 Q: 5

Question	Answer	Marks	Guidance
(i)	Dolphins | Sharks 5 | 9 9 5 5 3 2 | 6 | 4 6 8 5 3 2 | 7 | 0 1 2 4 7 2 2 0 | 8 | 0 4 Key: 3|6|4 means 63 kg for Dolphins and 64 kg for Sharks	B1	Correct stem can be upside down, ignore extra values,
		B1	Correct Dolphin must be on LHS,
		B1	Correct Sharks on either LHS or RHS of back-to-back. Alignment ± half a space, no late entries squeezed in, no crossing out if shape is changed. Condone a separate RHS stem-and-leaf diagram
		B1FT	Correct single key for *their* single diagram, need both teams identified and 'kg' stated at least once here or in leaf headings or title.
		4	
(ii)	Median = 72 LQ = 65, UQ = 80,	B1	72<UQ<82 – 62<LQ<72
	IQR = 80 – 65	M1	nfww
	= 15	A1	SCB1 if M0 scored for LQ = 65 **and** UQ = 80
		3	

27. 9709_s19_ms_61 Q: 1

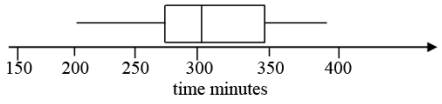
Question	Answer	Marks	Guidance
(i)	$\Sigma(t-120) = -25+6-3+15+0+5-6-1+16 = 7$	**M1**	Attempt to sum both $(t-120)$ and $(t-120)^2$ Correct ans using $\Sigma t - 9\times 120$ and $\Sigma (t-120)^2$ M1A1
	$\Sigma(t-120)^2 = 25^2+6^2+3^2+15^2+0^2+5^2+6^2+1^2+16^2$ $= 1213$	**A1**	Both correct, www, **SC** correct ans no working **B1B1**
		2	
(ii)	$\text{Var} = \frac{\Sigma(t-120)^2}{9} - \left(\frac{\Sigma(t-120)}{9}\right)^2 = \frac{their\,1213}{9} - \left(\frac{their\,7}{9}\right)^2$	**M1**	Using two coded values in correct formula including finding Σt from 7 etc
	= 134(.2)	**A1**	Correct answer **SC** if correct variance obtained by another method from raw data give **SCB1**
		2	

28. 9709_s19_ms_61 Q: 4

Question	Answer	Marks	Guidance
	Median Maths = 40	**M1**	Indication of finding medians, such as mark on graph or reference marks to 700 pupils, condone poor terminology such as 'mean'
	Median English = 55	**A1**	Both values correct, condone 54<English<56 but 54, 56 get A0
	Median of English is larger than median of Maths	**B1**	Correct statement, median must be referenced within answer. No credit if statement references 'means'
	Range Maths is 100 or IQ range Maths = 80 – 12 = 68	**M1**	Evidence of finding either both ranges or both IQ ranges i.e. see a minus
	Range English is 60 or IQ range English = 62 – 42 = 20	**A1**	Both ranges or IQR correct
	Maths marks have more spread then English marks	**B1**	Correct conclusion. Accept standard deviation but must see some figures
		6	

29. 9709_s19_ms_62 Q: 6

Question	Answer	Marks	Guidance
(i)	Advantage: comment referring to spread or range or shape	**B1**	Comments referring to quartiles, IQR, Range, median, shape, skewness, data distribution, spread score B1 Any comments with reference to mean or standard deviation or any other 'disadvantage' will score B0 Comments referring to '5-value plot', comparison with another data set, overview or ease of drawing/plotting/reading require an appropriate advantage statement.
	Disadvantage: comment referring to limited data information provided	**B1**	Comments referring to no individual data, no information about the number of values, unable to calculate mean, standard deviation, variance and mode score B1 Any comments with reference to median, shape or any other 'advantage' will score B0 Comments referring to 'size of data set' or 'average' require an appropriate disadvantage statement. Comments referring to outliers are ignored in all cases (as outliers are not in the syllabus content) unless supported by an appropriate advantage / disadvantage statement. If comments not clearly identified, assume first comment is the advantage.
		2	

Question	Answer	Marks	Guidance
(ii)	Not mean as data skewed by one large value	**B1**	Comment which identifies 768 (or 'a very large number') as the problem. Condone the use of 'outlier'
	Not mode as frequencies all the same	**B1**	Comment which indicates that no mode exists (e.g. all the data is different, there is no repeated number, all the values are different)
	Median	**B1**	Median identified as choice, dependent upon statements for mean and mode being given, even if incorrect or very general.
	SC: Mean is identified as most suitable		
	Not mode as frequencies all the same	**SCB1**	Comment which indicates that no mode exists
	Not median as not all values used	**SCB1**	Comment which indicates limitation of median e.g. median is not in middle of range.
		3	
(iii)(a)	LQ = 256 or 256.5 Med = 280 UQ = 329 Min 190 max 375  	**B1**	Median, UQ and LQ values seen, may not be identified or identified correctly. (Not read from box plot unless value stated)
		B1	**FT** Median and quartiles plotted in box on graph, linear scale
		B1	Correct end points, whiskers from ends of box but not through box, not at top or bottom of box
		B1	Uniform scale from 190 to 375 (need at least 3 linear identified points min) and labelled 'time' and 'minutes' (can be in title) **No time axis or time axis with no scale attempt,** Max B1B0B0B0
		4	

Question	Answer	Marks	Guidance
(iii)(b)	IQR = *their* 329 – *their* 256 = 73 or 72.5	**B1**	**FT Must** follow through only from ***their*** stated values (condone if correct quartiles stated here), not reading from graph.
		1	

30. 9709_s19_ms_63 Q: 7

Question	Answer	Marks	Guidance
(i)	Thaters School | | Whitefay Park School 8 | 3 8 3 | 4 | 5 7 8 8 7 6 4 2 | 5 | 3 6 6 6 2 1 | 6 | 1 4 6 9 5 | 7 | 3 5 8 | 8 | 3 Key 8 | 4 | 5 represents 48 minutes for Thaters School and 45 minutes for Whitefay Park School.	**B1**	Correct stem can be upside down, ignore extra values,
		B1	Correct Thaters School labelled on left, leaves in order from right to left and lined up vertically, no commas
		B1	Correct Whitefay Park School labelled on same diagram on right hand side in order from left to right and lined up vertically, no commas
		B1	**FT** Correct key for *their* diagram, need both teams identified and 'minutes' stated at least once here or in leaf headings or title. **SC** If 2 separate diagrams drawn, **SCB1** if both keys meet these criteria
		4	
(ii)	LQ = 50 UQ = 61.5	**B1**	Both quartiles correct
	IQ range = 61.5 – 50 = 11.5	**B1**	**FT** 61 ⩽ UQ ⩽ 62 – 48 ⩽ LQ ⩽ 52
		2	
(iii)	$\Sigma(x - 60)^2 = (-15)^2 + (-13)^2 + (-7)^2 + (-4)^2 + (-4)^2 + 1^2 + 4^2 + 6^2 + 9^2 + 13^2 + 23^2 + 15^2 + 18^2$	**M1**	Summing squares with at least 5 correct unsimplified terms
	= 1856	**A1**	Exact value
		2	

Question	Answer	Marks	Guidance
(iv)	Var = mean of coded squares – (coded mean)^2 $= \frac{\Sigma(x-60)^2}{13} - \left(\frac{\Sigma(x-60)}{13}\right)^2$	M1	Using two coded values in correct formula (variance or sd)
	Var $= \frac{their\,1856}{13} - \left(\frac{46}{13}\right)^2$ $= 130$	A1	Correct answer SC if correct variance obtained by another method give SCB1
		2	

31. 9709_w19_ms_61 Q: 3

Question	Answer	Marks	Guidance
(i)	$\Sigma x = 60 \times 20 \quad = 1200$	B1	
	$\frac{\Sigma x^2}{20} - 60^2 = 4^2$	M1	Correct variance formula used, condone = 4
	$\Sigma x^2 = 3616 \times 20 \quad = 72320$	A1	Exact value
		3	

Question	Answer	Marks	Guidance
(ii)	$\Sigma x = 1200 + 550 = 1750$ $\Sigma x^2 = 72320 + 40500 = 112800$	M1	Summing both values of Σx and Σx^2
	Mean $= \frac{their\,1750}{30} = 58.3$	B1FT	FT *their* 1750 (not 550 or 1200)/*their*(20+10), accept unsimplified
	Variance $= \frac{their\,112820}{30} - \left(\frac{their\,1750}{30}\right)^2 \quad (= 357.89)$	M1	substitute *their* Σx and Σx^2 into correct variance formula
	s.d. = 18.9	A1	
		4	

32. 9709_w19_ms_61 Q: 5

Question	Answer	Marks	Guidance
(i)	156 – 55 = 99	B1	98 ⩽ answer < 100
		1	
(ii)	90% of 160 = 144	M1	144 seen, may be marked on graph
	(L =) 22	A1	
		2	
(iii)	Median = 15.6 UQ = 18.8, LQ = 12.7	B1	15.5 < median < 15.8
	IQR = 18.8 – 12.7	M1	18.5 < UQ < 19 – 12.5 < LQ < 13
	6.1	A1	6.0 ⩽ IQR ⩽ 6.2
		3	
(iv)	The Median higher for Ransha (1st set of data)	B1	Any correct comparison of central tendency, must mention median
	IQR lower for Ransha (1st set of data)	B1	Any correct comparison of spread, must refer to IQR
		2	

33. 9709_w19_ms_62 Q: 1

Question	Answer	Marks	Guidance
(i)	Median = 51 UQ = 57.5, LQ = 40	B1	
	IQR = UQ – LQ	M1	$55 \leqslant UQ \leqslant 62 - 38 \leqslant LQ \leqslant 45$
	17.5	A1	NFWW
		3	
(ii)	Result will be disproportionately affected by 110	B1	Affected by an extreme/large value There is a large outlier …contains outliers such as 110… Not 'mean affected by extreme values'
		1	

34. 9709_w19_ms_62 Q: 3

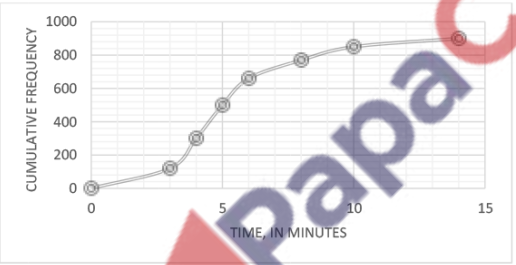
Question	Answer	Marks	Guidance
(i)	0.5 2.4 3 1.4 0.4	M1	At least 3 frequency densities calculated (frequency ÷ class width) e.g. $\left(\frac{10}{20}, \frac{10}{19} \text{ or } \frac{10}{19.5}\right)$ may be read from graph using *their* scale, 3SF or exact
	All heights correct on graph.	A1	
	Bar ends of 9.5, 29.5, 39.5, 59.5, 89.5	B1	
	Axes labelled: Frequency density (fd) and speed/km h^{-1} (or appropriate title). Linear scales $9.5 \leqslant$ horizontal axis $\leqslant 89.5$, $0 \leqslant$ vertical axis $\leqslant 3$, 5 bars with no gaps	B1	
		4	

Question	Answer	Marks	Guidance
(ii)	$\frac{19.5\times10+34.5\times24+44.5\times30+54.5\times14+74.5\times12}{\textit{their}\ 90}$ $=\frac{195+828+1335+763+894}{90}$ $=\frac{4015}{90} \text{ or } \frac{803}{18}$	M1	Uses at least 4 midpoint attempts (e.g. 19.5 ± 0.5). Allow unsimplified expression.
	$44\frac{11}{18}$ or 44.6 (km h^{-1})	A1	Final answer not an improper fraction NFWW
		2	

35. 9709_w19_ms_63 Q: 5

Question	Answer	Marks	Guidance
(i)	Correct labels and scales	**B1**	Axes labelled 'cumulative frequency' (or cf) and 'time (or t) [in] min(utes)', linear scales from 0 to 90 and 0 to 200 with at least 3 values marked on each axis.
	7 correctly plotted points above upper boundaries joined in a curve or line segments	**B1**	(0, 0); (10, 16); (20, 50); (30, 106); (50, 146); (70,176); (90,200)
		2	
(ii)	29	**B1**	$28 \leqslant$ median $\leqslant 30$
		1	
(iii)	120 seen	**M1**	For seeing 120 in a calculation or marked on the graph
	37	**A1FT**	$36 \leqslant$ Ans $\leqslant 39$ or FT from *their* graph **SC1** unsupported answer in range
		2	
(iv)	Frequencies 16 34 56 40 30 24	**B1**	Seen. Allow unsimplified
	Est. Mean = $\frac{5\times16+15\times34+25\times56+40\times40+60\times30+80\times24}{200}$	**M1**	At least 4 correct midpoints (5, 15, 25, 40, 60, 80) used in a calculation
	$\frac{7310}{200}$	**M1**	Summing products of *their* 6 mid-points (not lower or upper bound or class width) × *their* frequencies / 200 (or *their* $\sum f$), unsimplified
	36.55	**A1**	Accept 36.6
		4	

36. 9709_m18_ms_62 Q: 1

Question	Answer	Marks	Guidance
		**M1 A1**	Attempt to plot cumulative frequencies at ucb and all points joined between $(3,y_1)$ and $(14,y_2)$. Cf table not required. Linear scales starting at (0,0) and axes labelled cf and time in mins, all points correct; (allow straight lines or curves)
	t: 0, 3, 4, 5, 6, 8, 10, 14 cf: 0, 120, 300, 500, 660, 770, 850, 900	**M1**	450 seen in median attempt on increasing CF graph (independent);
	Median value: 4.8 (minutes)	**A1 FT**	Correct ($4.7 \leqslant m < 4.9$) or FT from reading their increasing graph at cf = 450
		4	

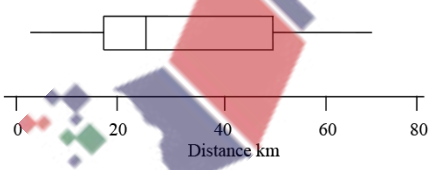
37. 9709_m18_ms_62 Q: 5

Question	Answer	Marks	Guidance
(i)	$24.25n - 20n = 136$ **Or** $\frac{136}{n} + 20 = 24.25$	**M1**	Unsimplified correct equation
	$n = 32$	**A1**	
		2	
(ii)	Using coded information: Variance = $\frac{2888}{32} - \left(\frac{136}{32}\right)^2$	**M1**	unsimplified expression for variance
	= 72.1875 = 72.19	**A1**	accept answers 72.2 SOI
	Using uncoded information: Variance = $\frac{\Sigma x^2}{32} - 24.25^2$ Equate with 72.1875 to give	**M1**	Equate two expressions for variance and solve
	$\Sigma x^2 = 21128$	**A1**	
		4	

38. 9709_s18_ms_61 Q: 1

Question	Answer	Marks	Guidance
	$\Sigma(x - 10) = 186 - 12 \times 10 = 66$	**B1**	Correct answer
	$\frac{\Sigma(x-10)^2}{12} - \left(\frac{\Sigma(x-10)}{12}\right)^2 = 4.5^2$	**M1**	Consistent substituting in the correct coded variance formula OR Valid method for Σx^2 then expanding $\Sigma(x - 10)^2$, 3 terms with at least 2 correct
	$\Sigma(x - 10)^2 = 606$	**B1**	Correct answer
		3	

39. 9709_s18_ms_61 Q: 2

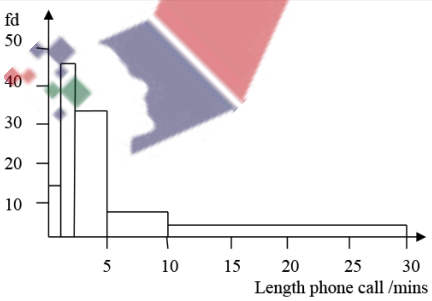
Question	Answer	Marks	Guidance
(i)	LQ = 18, Median = 25, UQ = 50  	**B1**	median correct
		B1	LQ and UQ correct
		B1	Quartiles and median plotted as box graph with linear scale min 3 values
		B1ft	Whiskers drawn to correct end points with linear scale, not thr' box, not joining at top or bottom of box. Ft their UQ and LQ. Whiskers must be with ruler If scale non-linear or non-existent SCB1if all 5 data values (quartiles and end points) have values shown and all are correct numerically and fulfil the 'box' and 'whiskers ruled line' requirements
		B1	Label to include 'distance or travelled' and 'km,' allow 'total km', linear scale, numbered at least 5 – 70.
		5	

Question	Answer	Marks	Guidance
(ii)	1.5 × IQR = 48 **Method 1** LQ – 48 = –ve, (i.e. < 0) UQ + 48 = 98 (i.e. > 70)	**M1**	Attempt to find 1.5 × their IQR and add to UQ **or** subt from LQ
	hence no outliers	**A1**	Correct conclusion from correct working, need both ends. No need to state comparisons.
	Method 2 LQ – 5 = 13 (< 48) 70 – UQ = 20 (< 48)	**M1**	Compare their 1.5 × IQR (= 48) > gap (20) between UQ and max 70 **or** LQ and min 5
	Hence no outliers	**A1**	Correct conclusion from correct working, need both ends. No need to state comparisons
		2	

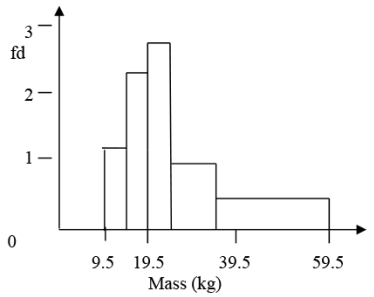
40. 9709_s18_ms_62 Q: 1

Question	Answer	Marks	Guidance
(i)	38	**B1**	
		1	
(ii)	Median = 38.5	**B1**	CAO
	IQR = 40 – 38	**M1**	39 < UQ < 45 – 36 < LQ ⩽ 38
	= 2	**A1**	If M0 awarded **SCB1** for **both** UQ = 40 or 40.5 **and** LQ = 38 or 37.75 seen
		3	

41. 9709_s18_ms_62 Q: 5

Question	Answer	Marks	Guidance
(i)	$a = 40$	**B1**	
		1	
(ii)	Mean = $\frac{0.5\times14+1.5\times46+3.5\times102+7.5\times\textit{their}\,40+20\times40}{242}$ $= \frac{1533}{242}$	**M1**	Numerator: 5 products with at least 3 acceptable mid-points × appropriate frequency FT **(i).** Denominator: 242 CAO $\frac{1533}{242}$ implies M1, but if **FT** an unsimplified expression required
	$= 6\frac{81}{242}$ or 6.33	**A1**	CAO (6.3347… rounded to 3 or more SF)
		2	
(iii)	fd = 14, 46, 34, ($\frac{\textit{their (i)}}{5}$ =) 8, 2	**M1**	Attempt at fd [f/(attempt at cw)] or scaled freq
		**A1FT**	Correct heights seen on diagram with linear vertical scale from (x, 0) **FT** their $\frac{a}{5}$ only
		B1	Correct bar widths (1:1:3:5:20) at axis, visually no gaps, with linear horizontal scale from ($0, y$), first bar starting at (0,0)
		B1	Labels (time, mins, and fd(OE) seen, some may be as a title) and a linear scale with at least 3 values marked on each axis. (Interval notation not acceptable)
		4	

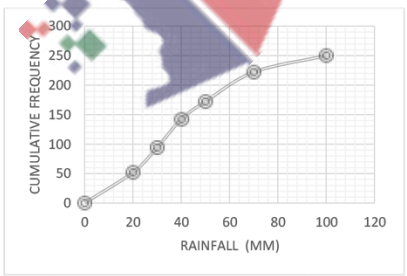
42. 9709_s18_ms_63 Q: 1

Question	Answer	Marks	Guidance
(i)	15–19 (kg) cao	B1	kg not necessary; condone 14.5 – 19.5
	Total:	1	
(ii)	fd = 1.2, 2.4, 2.8, 1, 0.32  	M1	Attempt at fd [f/(attempt at cw)] or scaled freq (may be implied by 4 correct)
		A1	Correct heights seen on diagram with linear vertical scale from (x, 0)
		B1	Correct bar widths (1:1:1:2:5) visually no gaps with linear horizontal scale from (9.5,y) and first bar starting at (9.5, y)
		B1	Histogram, using attempted fds, with labels (mass, kg and fd seen) and at least 3 linearly spaced values on each axis. Horizontal axis must range from at least 9.5 to 59.5 If horizontal axis clearly starts from zero, either a break in the scale must be indicated or the scale must be linear from zero.

43. 9709_s18_ms_63 Q: 4

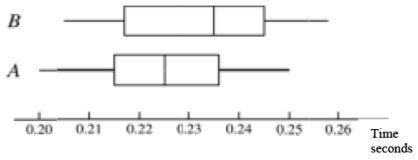
Question	Answer	Marks	Guidance
(i)	Mean = (30 × 1500 + 21 × 2400)/51	M1	Multiply by 30 and 21, summing and dividing total by 51 $\left(\frac{45000+50400}{51}\right)$
	= 1870 (1870.59)	A1	correct answer (to 3sf)
	Total:	2	
(ii)	$230^2 = \frac{\Sigma x_F^{\,2}}{30} - 1500^2$ so $\Sigma x_F^{\,2} = 69\,087\,000$	M1	One correct substitution into a correct variance formula
		A1	Correct $\Sigma x_F^{\,2}$ (rounding to 69 000 000 2sf)
	$160^2 = \frac{\Sigma x_L^{\,2}}{21} - 2400^2$ so $\Sigma x_L^{\,2} = 121\,497\,600$	A1	Correct $\Sigma x_L^{\,2}$ (rounding to 121 000 000 3sf)
	New var = $\frac{69087000+121497600}{51} - 1870.588^2 = 237\,853$	M1	using '$\Sigma x_F^{\,2}$'+ '' $\Sigma x_L^{\,2}$ dividing by 51 and subtracting 'i' squared. (Correct '$\Sigma x_F^{\,2}$' + '' $\Sigma x_L^{\,2}$ = 190 584 600)
	New sd = 488	A1	Correct answer accept anything between 486 and 490
	Total:	5	

44. 9709_w18_ms_61 Q: 6

Question	Answer	Marks	Guidance
(i)	 	B1	Appropriate linear scales starting at (0,0), axes labelled cf and Rainfall, mm
		B1	Correct graph, points plotted at ucb, allow straight lines or curve
		2	
(ii)		M1	Read off from increasing graph at cf = 150
	42	A1	Correct answer ($41 \leqslant r \leqslant 43$)
		2	

Question	Answer	Marks	Guidance
(iii)	Frequencies 52, 42, 48, 30, 50, 28	**B1**	Correct frequencies
	Mean age = $(10\times52+25\times42+35\times48+45\times30+60\times50+85\times28)/250$	**B1**	Correct midpoints (allow one error)
	$=9980/250$	**M1**	Using $\Sigma fx/250$ with mid-points attempt, not cf, cw, lb, ub
	$= 39.9(2)$ oe	**A1**	Correct answer
	Variance = $10^2\times52+25^2\times42+35^2\times48+45^2\times30+60^2\times50+85^2\times28)/250-\text{mean}^2$ $= 539.59$	**M1**	Attempt at variance using their midpoints and their mean
	$\sigma = 23.2$	**A1**	Correct answer for sd
		6	

45. 9709_w18_ms_62 Q: 2

Question	Answer	Marks	Guidance
(i)	median = 0.225; LQ = 0.215: UQ = 0.236	**B1**	Correct median (Q_2)
	IQR = 0.236 – 0.215	**M1**	$0.232 < \text{UQ } (Q_3) < 0.238 - 0.204 < \text{LQ } (Q_1) < 0.219$
	= 0.021	**A1**	www Omission of all decimal points **MR-1** If M0 awarded **SCB1** for both LQ = 0.215: UQ = 0.236 seen
		3	
(ii)		**B1**	Linear scale between 0.20 to 0.26 (condone omission of 0.26) axis labelled (time and seconds), at least one box plot attempted, no lines through boxes, whiskers not at corner of boxes
		B1 ft	Labelled correct graph for A, (ft their median/quartiles), condone lines through boxes, whiskers at corner of boxes
	<table><tr><td>**A**</td><td>0.200</td><td>0.215</td><td>0.225</td><td>0.236</td><td>0.250</td></tr><tr><td>**B**</td><td>0.205</td><td>0.217</td><td>0.235</td><td>0.245</td><td>0.258</td></tr></table>	**B1**	Labelled correct graph for B, condone lines through boxes, whiskers at corner of boxes **SC** If B0B0 scored because graphs not labelled/labels reversed SCB1 if both 'correct' Penalty **MR-1** if graphs plotted on separate axes unless both scales align exactly.
		3	

46. 9709_w18_ms_62 Q: 5

Question	Answer	Marks	Guidance
(i)	$\frac{15.5\times12+910}{12+20}$	M1	Unsimplified total age divided by *their* total members (not 12, 20 or 2)
	=34.25 or 34¼ (years)	A1	Correct exact answer (isw rounding), oe (34 years 3 months)
		2	
(ii)	Considering Juniors: variance = $\frac{\Sigma x^2}{12}-15.5^2=1.2^2$	M1	$\frac{\Sigma x^2}{k}-15.5^2=1.2^2$, $k=12$ or 20
	$\Sigma x^2=2900.28$	A1	Answer wrt 2900
	Considering whole group: $\Sigma z^2=\Sigma x^2+\Sigma y^2=2900.28+42850=45750$ Variance = $\frac{\Sigma z^2}{32}-\mu^2=\frac{\textit{their}\,45750}{12+20}-(\textit{their}\,34.25)^2$ (= 256.63)	M1	*Their* 45750 > 42850 (not 85700 or rounding to 1.8×10^9) in correct variance or std deviation formula (Σx^2 and addition may not be seen)
	s d = 16.0(2)	A1	Correct final answer, condone 16.03
		4	

47. 9709_w18_ms_63 Q: 7

Question	Answer	Marks	Guidance
(i)	Anvils | Brecons 8 | 15 9 5 | 16 | 6 5 3 2 0 | 17 | 0 1 2 2 8 4 1 0 | 18 | 1 2 3 3 6 | 19 | 2 Key: 5|16|6 means 165 cm for Anvils and 166 cm for Brecons	B1	Correct stem, up or down
		B1	Correct Anvils labelled on left, leaves in order from right to left and lined up vertically, no commas
		B1	Correct Brecons labelled on same diagram on right hand side in order from left to right and lined up vertically, no commas
		B1	Correct key, not split, both teams, at least one with cm
		4	
(ii)	Median = 173	B1	Correct median (or Q2)
	LQ = 169; UQ = 181 IQR = 181 – 169	M1	Either UQ = 181 ± 4, or LQ = 169 ± 4 and evaluating UQ – LQ
	= 12	A1	Correct answer from 181 and 169 only
		3	

Question	Answer	Marks	Guidance
(iii)	$\Sigma x = 1923 + 166 + 172 + 182$ (= 2443) $\Sigma x^2 = 337221 + 166^2 + 172^2 + 182^2$ (= 427485)	**M1**	Correct unsimplified expression for Σx and Σx^2, may be implied
	Mean $= \frac{\Sigma x}{14} = \frac{2443}{14} = 174.5$	**M1**	Correct unsimplified mean
	Variance $= \frac{\Sigma x^2}{14} - \left(\frac{\Sigma x}{14}\right)^2 = \frac{427485}{14} - \left(\frac{2443}{14}\right)^2$	**M1**	Correct unsimplified variance using 14, their Σx and their Σx^2, not using 1923 and/or 337221
	S d = 9.19	**A1**	Correct answer
		4	

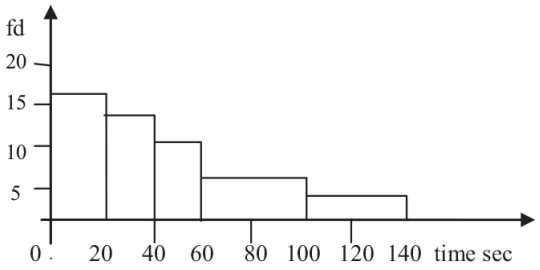
48. 9709_m17_ms_62 Q: 1

Question	Answer	Marks	Guidance
	1.6 –1.5 2.3 1.4 –0.6 –0.9 2.5 1.9 2.4 1.9 2.8 1.0	**M1**	Subtracting 1760, allow max 2 slips
	Mean = 1.23	**A1**	
	sd = 1.39	**A1**	
	Mean of x = 1761.23 , sd of x = 1.39	**A1**√	ft their coded mean and sd.
			SR B1 correct mean and sd without use of coded process
	Total:	**4**	

49. 9709_s17_ms_61 Q: 1

Question	Answer	Marks	Guidance
(i)	*EITHER:* $\frac{\sum x}{30} - k = \frac{315}{30} = 10.5$	**(M1**	Dividing 315 by ±30 and + or – from 50.5 need both and no more
	$k = 5.5 - 10.5 = 40$	**A1)**	Correct answer from correct working
	OR: $\sum x = 50.5 \times 30 = 1515$, $1515 - 30k = 315$	**(M1**	Mult by 50.5 by 30 and + or – 315 and dividing by ±30 need all these
	$k = 40$	**A1)**	Correct answer from correct working. 1200 gets **M0**
	Total:	**2**	
(ii)	*EITHER:* var = 4022/30–10.5^2(=23.817)	**(M1**	Subst in correct coded variance formula
	sd = 4.88	**A1)**	
	OR: $\sum x^2 - 2(40)\sum x + 30(40)^2 = 4022$, $\sum x^2 = 77222$ Var = 77222/30 – 50.5^2 (= 23.817)	**(M1**	Expanding with ± 40Σx and ± 30(40)^2 seen
	sd = 4.88	**A1)**	
	Total:	**2**	

50. 9709_s17_ms_61 Q: 4

Question	Answer	Marks	Guidance
(i)	fd 16, 14, 11, 505, 2.5	M1	Attempt at fd (must be at least 3 freq/cw) – may be implied by graph
		A1	Correct heights seen on graph i.e. must see a gap for fd = 2.5 etc.
		B1	Correct end points of bars and correct widths
		B1	labels fd, sec. Time can be optional. Linear axes, condone $0 \leqslant t < 20$ etc.
	Total:	**4**	

Question	Answer	Marks	Guidance
(ii)	$(10 \times 320 + 30 \times 280 + 50 \times 220 + 80 \times 220 + 120 \times 100) / 1140$	M1	using $\Sigma fx / n$ with mid-point attempt ±0.5, not ends not class widths
	$= 45.8$	A1	
	Total:	**2**	

51. 9709_s17_ms_62 Q: 1

Question	Answer	Marks	Guidance
(i)	$4 \times 5.5 + 3x + 90 = 8 \times 29$	M1	An expression to work out total cost of individual items = 8 × mean, x may be implied.
	$112 + 3x = 232$ $x = 40$	A1	Correct complete unsimplified expression / calculation
	(Cost = $)40	A1	Units not required
	Total:	**3**	
(ii)	sd = 0 so all cost the same	M1	Must see comment interpreting sd = 0, OE
	shirts cost 4 × $26 = $104 AG	A1	See 4 × $26, $130 – $26 OE. Must have a final value of $104 stated
	Total:	**2**	

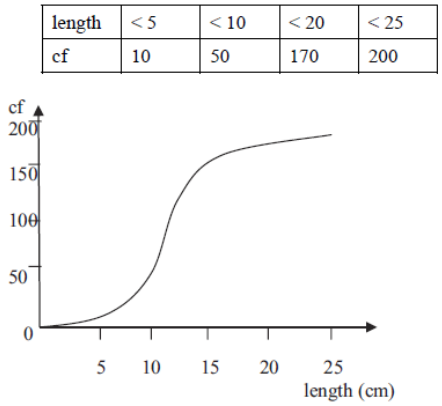
52. 9709_s17_ms_62 Q: 2

(i)	med = 3.2	B1	Accept 3.2 ± 0.05
	UQ = $3.65 \leqslant$ uq $\leqslant 3.7$ LQ = $2.55 \leqslant$ lq $\leqslant 2.6$	M1	UQ – LQ, UQ greater than *their* 'median', LQ less than *their* 'median'
	IQR = $1.05 \leqslant$ iqr $\leqslant 1.15$	A1	Correct answer from both LQ and UQ in given ranges
	Total:	**3**	
(ii)	$134 - 24 = 110$	B1	Accept $108 \leqslant n \leqslant 112$, n an integer
	Total:	**1**	

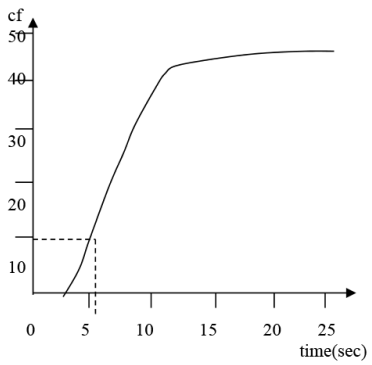
Question	Answer	Marks	Guidance
(iii)	200 – 12 = 188 less than length l	M1	188 seen, can be implied by answer in range, mark on graph.
	l = 4.5 cm	A1	Correct answer accept $4.4 \leqslant l \leqslant 4.5$
	Total:	**2**	

53. 9709_s17_ms_63 Q: 7

(i)	freq = fd × cw 10, 40, 120, 30	M1 A1	Attempt to multiply at least 3 fds by their 'class widths'
	Totals:	**2**	

Question	Answer	Marks	Guidance
(ii)		B1 B1 M1 A1	3 or more correct cfs **heights** on graph 10, 50, 170, 200 Labels correct cf and length(cm), linear scales from zero (allow 0.5 on horizontal axis) Attempt (at least three) at plotting at upper end points (either 5 or 5.5, 10 or 10.5 etc.) Starting at (0, 0) polygon or smooth curve increasing with plotted points at lengths 5, 10, 20 and 25
	Totals:	**4**	
(iii)	median = 14.2	B1	Median (accept 13.2 – 15.2)
	'18.5' – '10'	M1	Subt their LQ from their UQ if reasonable from their graph
	IQ range = 8.5	A1FT	Correct FT using LQ = 10 and UQ between 17.5 and 19.5
	Totals:	**3**	
(iv)	mean = (2.5×10 + 7.5×40 + 15×120 + 22.5×30) / 200	M1	Using mid points (± 0.5) and their frequencies from 7(i) in correct formula
	= 14	A1	
	Totals:	**2**	

54. 9709_w17_ms_61 Q: 2

Question	Answer	Marks	Guidance
(i)	Points (5.5,10), (8.5,25), (11.5,42), (16.5,46), (25.5,48)	B1	Correct cfs values seen listed, in or by table or on graph, 0 not required
		B1	Axes labelled "cumulative frequency" (or cf) and "time [or t etc.] (in) seconds (or sec etc.)". Linear scales – cf 0–48, time 2.5 – 25.5 (ignore <2.5 on time.) At least 3 values stated on each axis, but (0,0) can be implied without stating.
		B1	All points plotted accurately, (5, 10) etc. scores **B0**. Curve or line segments drawn starting at (5.5,10) and passing within '1 scale unit' vertically and horizontally of plotted points
		3	

Question	Answer	Marks	Guidance
(ii)	48 – 35 = 13 $t =$ 6.5 sec	M1	Subt 35 (checked ±1 mm on graph) from 48 or 50,
		A1	$6 \leqslant$ Ans $\leqslant 7$
		2	

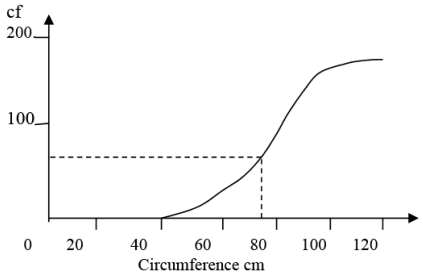
55. 9709_w17_ms_61 Q: 4

Question	Answer	Marks	Guidance
(i)	$\frac{(48.7\times12+38.1\times7)}{19}$	M1	Accept unsimplified (may be separate calculations)
	$= 44.8$	A1	
		2	
(ii)	$7.65^2 = \frac{\Sigma x^2}{12} - 48.7^2 \quad \Sigma x^2 = 29162.55$	M1	Substitution in one correct variance formula
	$4.2^2 = \frac{\Sigma y^2}{7} - 38.1^2 \quad \Sigma y^2 = 10284.75$	A1	One Σx^2 or Σy^2 correct (can be rounded to 4sf))
	Combined var $= \frac{(29162.55+10284..75)}{19} - 44.79^2$ $= \frac{39447.3}{19} - 44.79^2$	M1	Using their Σx^2 and Σy^2 and their **4(i)** in the variance formula
	Combined $\sigma = 8.37$ or 8.36	A1	
		4	

56. 9709_w17_ms_62 Q: 1

Question	Answer	Marks	Guidance
	EITHER: $(\Sigma x =)\ 11.5n = 27 + 10n$	(M1	Expanding brackets and forming a three term equation involving 27 and at least one term in n, without x
		M1	$10n$ or $11.5n$ seen in expression without x ($1.5n = 27$ implies **M2**)
	$n = 18$	A1)	
	OR: $11.5 = \frac{27}{n} + 10$	(M1	Dividing coded sum by n and forming a three term equation involving 11.5 and at least one term in n, without x
		M1	$27/n$ seen in expression without x ($1.5 = \frac{27}{n}$ implies **M2**)
	$n = 18$	A1)	
		3	

57. 9709_w17_ms_62 Q: 2

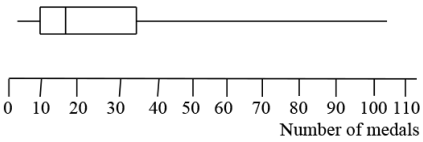
Question	Answer	Marks	Guidance
(i)	points (50, 14), (80, 62), (100, 132), (120, 140)	B1	Correct cfs values seen listed, in or by table or on graph, 0 not required
		B1	Axes labelled 'cumulative frequency' (or cf) and 'circumference [or cir or c etc.] (in) cm'. Linear scales – c.f. 0–140 circumference 40–120 (ignore <40 on circ.) At least 3 values stated on each axis, but (0,0) can be implied without stating.
		B1	All points plotted accurately
		3	
(ii)	$140 - 54 = 86$	M1	Finding correct value from graph (checked ±1 mm) or linear interpolation. Subtraction from 140 can be implied
	Percentage = 61.4%	A1	$60.5\% \leqslant \text{Ans} \leqslant 64.5\%$
		2	

58. 9709_w17_ms_63 Q: 2

Question	Answer	Marks	Guidance
	$\Sigma(x - 45) = 1218 - 20 \times 45 = 318$	B1	
	$\frac{\Sigma(x-45)^2}{20} - \left(\frac{\Sigma(x-45)}{20}\right)^2 = 4.2^2$	M1	Fully correct substitution in the correct coded variance formula with their $\Sigma(x - 45)$ **OR** valid method for $\Sigma x^2 = 74\,529$ ($4.2^2 = \frac{\Sigma x^2}{20} - \left(\frac{1218}{20}\right)^2$) and expanding $\Sigma(x-45)^2$ correctly $= \Sigma x^2 - 90\Sigma x + 20 \times 45^2 = '74\,529' - 90 \times 1218 + 40\,500 = 5409$
	$\Sigma(x - 45)^2 = 5409$	A1	
		3	

59. 9709_w17_ms_63 Q: 5

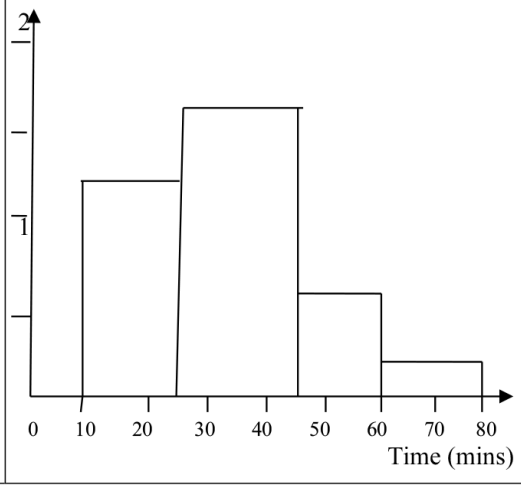
Question	Answer	Marks	Guidance
(i)	0 | 2 2 5 6 9 1 | 0 0 0 2 2 3 3 4 7 7 8 8 2 | 8 8 3 | 4 5 8 4 | 4 5 | 6 | 5 7 | 8 | 2 8 9 | 10 | 4 key 2 | 8 means 28 medals	B1	Stem, digits 5, 7, 9 can be missing here, can be upside down
		B1	All leaves in correct order increasing from stem, (5, 7 and 9 can be missing), condone commas
		B1	Reasonable shape, requires all values of the stem, only one line for each stem and leaves must be lined up. Can be upside down or sideways. No commas. Condone one 'leaf' error.
		B1	Correct key must state 'medals' or have 'medals' in leaf heading or title
		4	

Question	Answer	Marks	Guidance
(ii)	Med = 17 LQ = 10 UQ = 35  	B1	Median correct
		B1	LQ and UQ correct
		B1	Uniform scale from 2 to 104 (need 3 identified points min) and label including medals (can be in title)
		B1 FT	Correct box med and quartiles on diagram, FT their values
		B1	Correct end-whiskers from ends of box but not through box
		5	

60. 9709_m16_ms_62 Q: 1

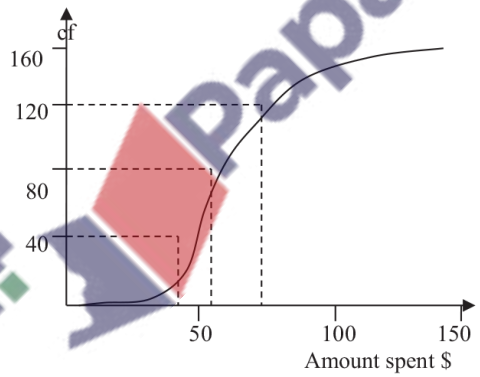
(i)	$\Sigma x = 862$	**B1** 1	Must be stated or replaced in (ii) Can see **(i)** and **(ii)** in any order
(ii)	$362/10 + a = 86.2$ $a = 50$	**M1** **A1** 2	86.2 ± 36.2 seen oe Correct answer, nfww

61. 9709_m16_ms_62 Q: 4

(i)	32	**B1**	1	
(ii)	freqs 0 18 32 9 4 fd 0 1.2 1.6 0.6 0.2 cf  	**M1** **A1** **B1** **B1**	 4	attempt at fd or scaled freq (at least 3 f/cw attempt) correct heights seen on diagram Correct bar ends Labels fd and time (mins) and linear axes or squiggle

(iii)	$(17.5 \times 18 + 35 \times 32 + 52.5 \times 9 + 70 \times 4)/63$ $= 2187.5/63 = 34.7$	**M1** **A1**	 2	$\Sigma fx/63$ where x is midpoint attempt not end pt or cw Correct answer

62. 9709_s16_ms_61 Q: 7

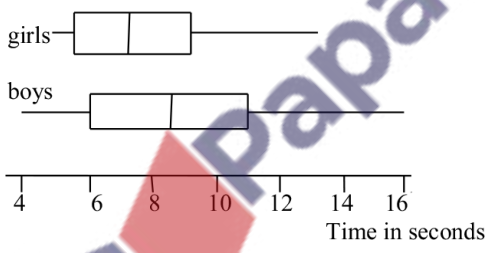
(i)	cf 16, 56, 104, 130, 160  	**M1** **B1** **M1** **A1**	 [4]	Attempt at cf table (up to 160) no graph needed accept %cf but give final linear scale minimum 0 to 160 and 0 to 120 Attempt to plot points at (30, 16), (50, 56), (70,104), (90, 130), (140, 160) up to 2 errors can have a polygon All points correct from their scale and joined up, with (0,0) as well
(ii)	median \$59 IQR = 82 – 43 = \$39	**B1√** **M1** **A1√**	 [3]	accept 57–60 or ft their graph if used lb, midpts instead of ub or assume linear interpolation. Subt a (sensible) LQ from a sensible UQ (generous) Ans ft need a cf graph and UQ 80–84, LQ

Question	Answer	Marks	Guidance
(iii)	160 – 149 = 11 OR 115 is mid pt of last interval so # of shoppers is 30/2 = 15 (can be implied)	M1 A1 [2]	41–46 Subtracting from 160 can be implied Correct answer accept 9–16
(iv)	mean = (15×16+ 40×40 +60×48+ 80×26 + 115×30)/160 = 10250/160 = $64.1= $64.1	M1 A1 [2]	Using $\Sigma xf/160$ with mid-points

63. 9709_s16_ms_62 Q: 5

(i)	Bronlea Rogate 6 3 0 4 5 7 7 7 4 3 1 0 1 3 5 6 8 8 7 5 4 2 1 2 3 3 6 3 2 3 4 5 4	B1 B1 B1 B1	Correct single stem Correct ordered leaves Bronlea Correct ordered leaves Rogate Correct overall shape
	Key 3| 1 | 5 represents 13 kph for Bronlea and 15 kph for Rogate	B1 [5]	Single key must have both towns and units consistent with their values
(ii)	median Bronlea = 23 km per hour IQ range Rogate = 23 – 7 = 16	B1 M1 A1 [3]	Units not necessary Subt their LQ <14 from their UQ>14 from Rogate leaf
(iii)	Rogate is less windy than Bronlea	B1 [1]	Not a comparison of a statistic but interpretation of information

64. 9709_s16_ms_63 Q: 2

(i)		B1 B1 B1 [3]	Labels 'time' **and** 'seconds', 'boys' and 'girls' on correct plots and scaled line One box and whisker all correct on graph paper – ignore boy or girl label Second box and whisker all correct (on graph paper and ignore boy/girl label) on SAME scaled line.
(ii)	girls smaller range or IQ range than boys /girls less spread out oe girls generally quicker than boys or girls median<boys median (not mean) oe boys almost symmetrical, girls +vely skewed oe	B1 B1 [2]	Any 2 comments – MUST be a comparison

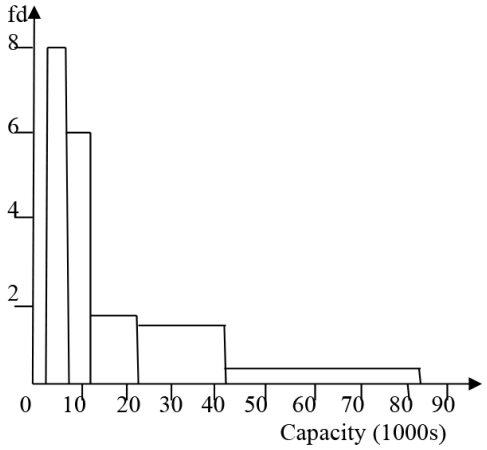
65. 9709_s16_ms_63 Q: 4

Qu	Answer	Marks	Guidance
(i)	1845/9 (= 205) c = 2205 - 205 = 2000	M1 A1	Accept (1845± anything)/ 9
	OR $\Sigma x = 2205 \times 9$ (= 19845) $\Sigma x - \Sigma c = 1845$ Σc = 19845 -1845 = 18000 c = 2000	M1 A1 [2]	For 2205×9 seen
(ii)	var = $\frac{477450}{9} - 205^2$ = 11025	M1 A1	For $\frac{477450}{9}$ – (their coded mean)^2
	OR var = $\frac{43857450}{9} - 2205^2$ = 11025	M1 A1 [2]	For their $\Sigma x^2/9 - 2205^2$ where Σx^2 is obtained from expanding $\Sigma(x-c)^2$ with $2c\Sigma x$ seen
(iii)	new total = 2120.5×10 = 21205 new price = 21205 – 19845 = 1360	M1 A1 [2]	Attempt at new total

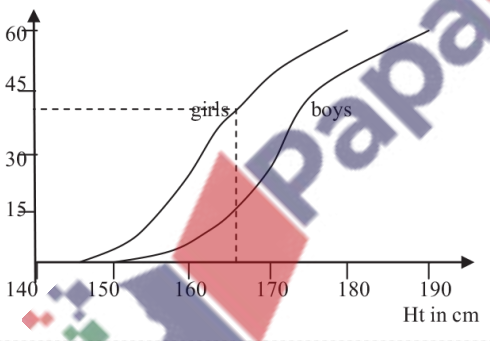
66. 9709_w16_ms_61 Q: 7

(i)	Factory A | | Factory B 3 | 1 5 8 9 | 4 | 2 4 7 8 9 9 8 8 7 4 3 0 | 5 | 1 4 6 8 5 3 1 1 1 | 6 | 4 Key: 9 | 4 | 2 represents 0.049g for factory A and 0.042 g for factory B	M1 B1 B1 B1 B1 [5]	Attempt at ordering factory B Correct stem Correct leaves factory A Correct leaves factory B Correct key need factory A and factory B and units
(ii)	median factory B = 0.048 g IQR = UQ – LQ = 0.055 – 0.04 = 0.015	B1 M1 A1 [3]	using their key i.e. 48, 0.48 etc or correct Subt their LQ from their UQ for factory B
(iii)	generally heavier in factory A Masses more spread out in factory B	B1 B1 [2]	oe must refer to context, e.g. mass

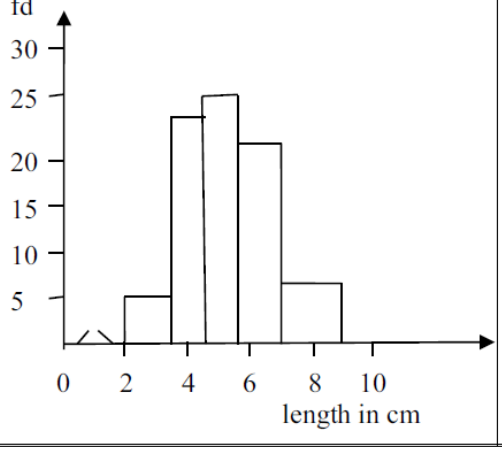
67. 9709_w16_ms_62 Q: 5

(i)	cw 5, 5, 10, 20, 40 fd 8, 6, 1.8, 1.7, 0.2  	**M1** **M1** **A1** **B1** **B1**	[5]	cw either 4 or 5 etc fd or scaled freq [f/their cw attempt] fd may be ÷ 1000 Correct heights seen accurately on diagram Correct bar ends, accurately plotted on axis Labels fd and capacity (thousands) Correct horizontal scale required. Vertical scale linear from 0
(ii)	(5×40+10×30+17.5×18+32.5×34+62.5×8)/130 = 2420/130 = 18.6 thousand	**M1** **A1**	[2]	Σfx/130 where x is mid point attempt (value within class, not end pt or cw)
(iii)	median group = 8 – 12 thousand LQ group = 3 – 7 thousand	**B1** **B1**	[2]	Thousands not needed

68. 9709_w16_ms_63 Q: 5

(i)	cf  	**B1** **B1** **B1** **B1**	[4]	Horizontal axis from min of 140 to 190 and vertical axis from 0 to minimum of 60 and two CF graphs on the same set of axes. Labels: CF; height (ht) in cm; girls; boys in correct places CF graph going through (150, 0) , (160, 20), (170, 43), (180, 55) and (190, 60) CF graph going through (140, 0), (150, 12), (160,33), (170,50), (180, 60) [and (190, 60)]
(ii)	42 (± 1) shorter than 165. (18(± 1))/60×100 = 30% (± 1.7%)	**M1** **M1** **A1**	[3]	Line or reading from 165 on their cf graph oe subtracting from 60
(iii)	can see which is taller; see which of boys or girls is more spread out	**B1**	[1]	any sensible comment in context

69. 9709_s15_ms_61 Q: 2

(i)	UQ 5.5 – 7.0 cm	**B1 [1]**	
(ii)	fd 5.33, 25, 28, 20.7, 6,  	**M1** **A1** **B1** **B1 [4]**	Attempt at fd or scaled freq [fr/cw] Correct heights seen on graph Correct bar widths no gaps Labels (fd and length/cm) and correct bar ends

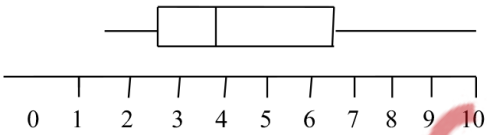
70. 9709_s15_ms_61 Q: 5

(i)	new mean $= \dfrac{9\times 7.1+18\times 5.2}{27}$ $= 5.83$	**M1** **A1 [2]**	Mult by 9 and 18 and dividing by 27 correct answer
(ii)	$1.45^2 =$ so $\dfrac{\Sigma x_t^2}{9} = 472.6125$ mm	**M1**	subst in a correct variance formula sq rt or not
		A1	correct Σx_t^2 (rounding to 470)
	$0.96^2 = \dfrac{\Sigma x_g^2}{18} - 5.2^2$ so $\Sigma x_g^2 = 503.3088$	**A1**	correct Σx_g^2 (rounding to 500)
	New sd^2 $\dfrac{472.6\ldots^2 + 503.3\ldots^2}{27} - 5.83\ldots^2 = 2.117$	**M1**	using $\Sigma x_t^2 + \Sigma x_g^2$, dividing by 27 and subt comb mean^2
	New sd = 1.46	**A1 [5]**	correct answer

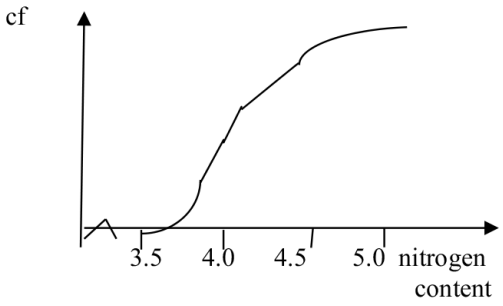
71. 9709_s15_ms_62 Q: 2

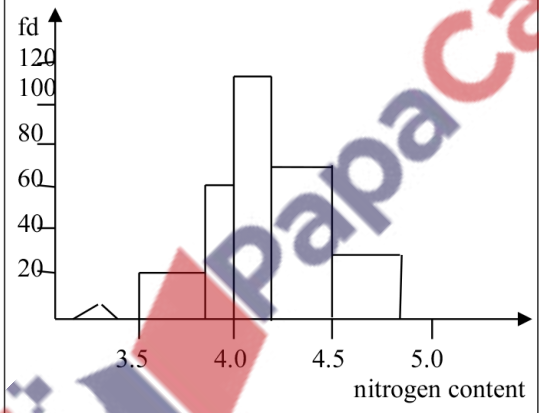
	mid points 13, 30.5, 40.5, 50.5, 73 Mean = $\frac{4\times13\ +24\times30.5\ +38\times40.5\ +34\times50.5\ +20\times73}{120}$ $=\frac{5500}{120}=45.8$ var = $\frac{4\times13^2+24\times30.5^2+38\times40.5^2+34\times50.5^2+20\times73^2}{120}$ $-(45.8\ldots)^2$ $=\frac{278620}{120}-45.8\ldots^2$ $=2321.8333-45.8\ldots^2$ sd = 14.9	M1 M1 A1 M1 A1	**5**	Attempt at midpoints at least 3 correct Using their midpoints i.e. cw, ucb, 1/2 cw and freqs into correct formula must be divided by 120 Correct answer from correct working Evaluating $\frac{\Sigma fx^2}{120}$ – their $\bar{x}^2$ must see their 45.8^2 subtracted allow cw etc Correct answer

72. 9709_s15_ms_62 Q: 3

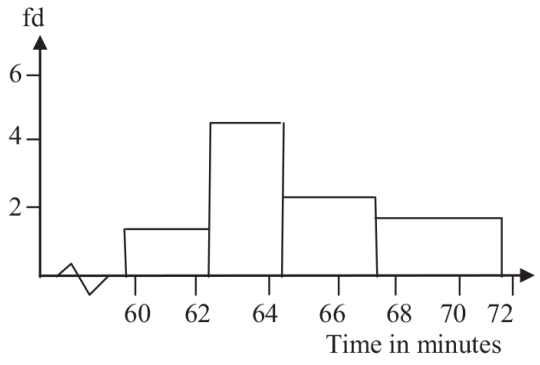
(i)	 	B1 B1 √ B1 B1	**4**	LQ = 2.6 med = 3.8– 3.85, UQ = 6.4– 6.6 Correct quartiles and median on graph ft linear from 2–10 End whiskers correct not through box Label need seconds and linear 2–10 axis or can have 5 values on boxplot no line provided correct
(ii)	1.5 × IQR = 1.5 × 3.8 = 5.7 LQ – 5.7 = –ve, UQ + 5.7 = 12.1 i.e. > 10 So no outliers AG	M1 A1	**2**	Attempt to find 1.5 × IQR and add to UQ or subt from LQ OR compare 1.5 × IQR with gap 3.6 between UQ and max 10 Correct conclusion from correct working need both

73. 9709_s15_ms_63 Q: 6

(i)		B1 M1 A1 **[3]**	Uniform axes cf and nitrogen content labelled, at least 0 to 70 and 3.5 to 4.8 seen 5 points plotted correctly on graph paper 3.5 | 3.8 | 4.0 | 4.2 | 4.5 | 4.8 0 | 6 | 18 | 41 | 62 | 70 All points correct and a reasonable curve (condone 1 missed point) or line segments.
(ii)	70 – their 55 = 15 = 21.4%	M1 A1 **[2]**	Subt a value > 41 from 70 (or n/70, n<29) Correct ans, accept 18.5 – 22
(iii)	median = 4.15	B1 **[1]**	Accept 4.1< median < 4.2, nfww

(iv)	nit cont: 3.5–3.8 | 3.8–4.0 | 4.0–4.2 | 4.2–4.5 | 4.5–4.8 fr: 6 | 12 | 23 | 21 | 8 fd: 20 | 60 | 115 | 70 | 26.7 	M1 M1 A1 B1 B1 **[5]**	Attempt at freqs, at least 3 correct, ignore labelling Attempt at fd as f/cw only at least 3 correct FT (Accept f/cw × k) Correct heights seen on graph (plot at 4.8,27 A0) Graph paper must be used (3 correct relative heights implies M1M1) Correct bar ends seen on graph – graph paper used Correct linear scale and labels.

74. 9709_w15_ms_61 Q: 3

(i)	$a = 9/cw$ $= 9/2 = 4.5$ $1.5 = b/4$ so $b = 6$	**M1** **A1** **A1** [3]	Using fd = f/cw Correct a Correct b
(ii)		**B1**√ **B1** **B1** [3]	Correct heights ft their b Correct widths, ie 3, 2, 3, 4 starting either 60 or 59.5 Labels fd, time or minutes and squiggle and bars from 59.5 to 71.5

75. 9709_w15_ms_62 Q: 1

	$\Sigma x - 100n = 216$ $2416 - 100n = 216$ $n = 22$ OR $\frac{2416}{n} = \frac{216}{n} + 100$ $n = 22$	**B1** **B1** **B1** 3 **B1** **B1** **B1**	$\Sigma x - 100n$ seen Subst 2416 for their Σx Correct answer $2416/n$ seen or $216/n + 100$ oe eg $\Sigma x/n - 100 = 216/n$ correct equation Correct answer

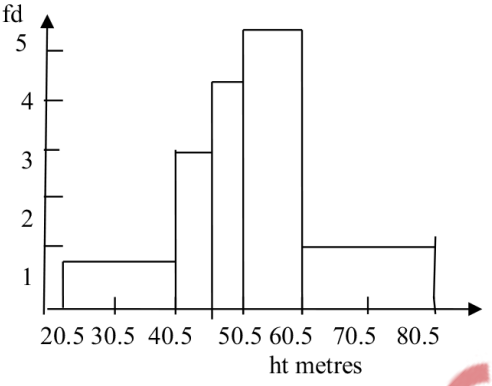
76. 9709_w15_ms_62 Q: 5

(i)	team *A* |	team *B* | | 7 | 5 7 9 4 4 2 | 8 | 2 3 4 6 9 8 7 6 1 | 9 | 4 5 6 9 7 4 0 | 10 | 1 8 6 5 | 11 | 1 3 5 2 | 12 | key 1 | 9 | 4 means 91 kg for team *A* and 94 kg for *B*	**B1** **B1** **B1** **B1** 4	Correct stem can be upside down, ignore extra values, allow 70, 80 etc with suitable numerical key Correct team *A* must be on LHS, alignment ± half a space, no late entries squeezed in, no crossing out if shape is changed Correct team *B* in single diagram can be either LHS or RHS Correct key or keys for their diagram/s, need both teams, at least one kg.
(ii)	LQ = 91 UQ = 109 IQ range = 18	**B1** **B1**√ 2	Both quartiles correct Correct IQR ft wrong quartiles, LQ < UQ, not 12 – 4 etc
(iii)	$\Sigma x_{15} = 1399$ $\Sigma x_{16} = 16 \times 93.9 = 1502.4$ New wt = 1502.4 – 1399 = 103 (103.4)	**M1** **M1** **A1** 3	Attempt at Σx_{15} for either team Mult 93.9 by 16 attempt Correct answer

77. 9709_w15_ms_63 Q: 1

	coded mean = 0.3 oe	**B1**		$\Sigma(t - 2.5) = 75$ B0 until ÷ 250
	sd $= \sqrt{\frac{96.1}{250} - (0.3)^2}$	**M1**		Subst in variance formula both terms coded
	$= 0.543$	**A1**	3	Correct answer
	Alt: $\Sigma(t-2.5)^2$ expanded	**Or**		
	$\Sigma t^2 = 2033.6$	**B1**		
	sd $= \sqrt{\frac{2033.6}{250} - 2.8^2}$	**M1**		Substituting their Σt^2 from expanded 3-term expression, 250 and 2.8 in variance formula
	$= 0.543$	**A1**	3	

78. 9709_w15_ms_63 Q: 6

(i)	fd 0.9, 3, 4.2, 5.2, 1.4	**M1**		Attempt at scaled freq [f/(attempt at cw)]
		**A1**		Correct heights seen on diagram Scale no less than 1cm to 1 unit
		B1		Correct bar widths visually no gaps
		B1	4	Labels (ht/metres and fd or freq per 20 m etc.) and end points at 20.5 etc. condone 2 end point errors, scale no less than 1cm to 5m for 20,30… unless clearly accurate, linear scale between 20.5 and 80

(ii)	$(30.5 \times 18 + 43 \times 15 + 48 \times 21 + 55.5 \times 52 + 70.5 \times 28)/134$	**M1**		Attempt at unsimplified, mid points (at least 4 within 0.5)
	$= \frac{7062}{134} = 52.701$	**M1**		Attempt at Σfx their mid points ÷ 134
		A1		Correct mean rounding to 53
	Var $= (30.5^2 \times 18 + 43^2 \times 15 + 48^2 \times 21 + 55.5^2 \times 52 + 70.5^2 \times 28)/134 - 52.701^2$	**M1**		Attempts at Σfx^2 their mid points ÷ their Σf – mean^2
	$= 392203.5/134 - 52.701^2 = 149.496$ sd = 12.2	**A1**	5	Correct answer, nfww

79. 9709_m22_ms_52 Q: 5

Question	Answer	Marks	Guidance
(a)	$^5C_1 \times {}^7C_4$	**M1**	$^7C_4 \times k$, k integer $\geqslant 1$ Condone $^5P_1$ for M1 only
	175	**A1**	
		2	

Question	Answer	Marks	Guidance
(b)	2B 1G 2A $^3C_2 \times {}^4C_1 \times {}^5C_2 = 120$ 2B 2G 1A $^3C_2 \times {}^4C_2 \times {}^5C_1 = 90$ 2B 3G $^3C_2 \times {}^4C_3 = 12$ 3B 1G 1A $^3C_3 \times {}^4C_1 \times {}^5C_1 = 20$ 3B 2G $^3C_3 \times {}^4C_2 = 6$	M1	$^3C_x \times {}^4C_y \times {}^5C_z$, $x + y + z = 5$, x,y,z integers $\geqslant 1$ Condone use of permutations for this mark
		B1	2 appropriate identified outcomes correct, allow unsimplified
		M1	Summing *their* values for 4 or 5 correct identified scenarios only (no repeats or additional scenarios), condone identification by unsimplified expressions
	[Total =] 248	A1	Note: Only dependent upon M marks
		4	
(c)	$8! \times 3! \times {}^5P_2$	M1	$8! \times m$, m an integer $\geqslant 1$ Accept $8 \times 7!$ for $8!$
		M1	$3! \times n$, n an integer > 1
		M1	$p \times {}^5P_2$, $p \times {}^5C_2 \times 2$, $p \times 20$, p an integer > 1 If extra terms present, maximum 2/3 M marks available
	4 838 400	A1	Exact value required
		4	

80. 9709_s22_ms_51 Q: 1

Question	Answer	Marks	Guidance
(a)	5!	M1	$k!$ where $k = 5$, 6 or 7 Condone $\times 1$ OE
	120	A1	
		2	
(b)	[Total no of ways =] $\frac{8!}{2!3!}$ [= 3360]	M1	$\frac{8!}{a!b!}$, $a = 1,2$ $b = 1,3$ $a \neq b$
	[With 3Es together =] $\frac{6!}{2!}$ [= 360]	M1	$\frac{6!}{c!}$, $c = 1,2$ seen in an addition/subtraction
	[With 3Es not together] = 3360 – 360	M1	$\frac{8!}{d!e!} - \frac{6!}{f!}$ where $d, f = 1,2$ & $e = 1,3$
	3000	A1	
		4	

81. 9709_s22_ms_51 Q: 2

Question	Answer	Marks	Guidance
(a)	$^{12}C_4 \times 2$	M1	$^gC_4 \times h$ $g = 12, 13$, $h = 1,2$
	990	A1	
	Alternative method for question 2(a)		
	[total – both on – neither on] $^{14}C_5 - (^{12}C_3 + {}^{12}C_5) = [2002 - 220 - 792]$	M1	$^kC_5 - (^aC_3 + {}^aC_5)$ $a = 12, 13$ and $k = 13, 14$
	990	A1	
		2	
(b)	[Mrs Lan plus] 2W 2M $^7C_2 \times {}^6C_2 = 315$ 3W 1M $^7C_3 \times {}^6C_1 = 210$ 4W $^7C_4 = 35$	M1	$^7C_r \times {}^6C_{4-r}$ for $r = 2$, 3 or 4
		B1	Outcome for one identifiable scenario correct, accept unevaluated
		M1	Add outcomes for 3 identifiable correct scenarios Note: if scenarios not labelled, they may be identified by seeing $^7C_r \times {}^6C_s$ $r + s = 4$ to imply r women and s men for both **B & M** marks only
	[Total =] 560	A1	
		4	

82. 9709_s22_ms_52 Q: 6

Question	Answer	Marks	Guidance
(a)	$\left[\frac{9!}{2!2!}=\right]$ 90 720	B1	
		1	
(b)	**Method 1** Arrangements Cs at ends – Arrangements Cs at ends and Os together		
	[Os not together =] $\frac{7!}{2!}-6!$ [= 2520 – 720]	M1	$\frac{w!}{2!}-y$, $w=6, 7$ y an integer. Condone $2\times\left(\frac{w!}{2!}\right)-y$.
		M1	$a-6!$ or $a-720$, a an integer resulting in a positive answer.
	1800	A1	
	Method 2 identified scenarios R ^ ^ ^ R		
	[Os not together =] $5!\times\frac{6\times5}{2!}=$	M1	$5!\times b$, b integer >1.
		M1	$c\times\left(\frac{6\times5}{2!}\text{ or }{}^6C_2\text{ or }\frac{{}^6P_2}{2!}\text{ or }15\right)$, c integer > 1.
	1800	A1	
		3	

Question	Answer	Marks	Guidance
(c)	CCO _ $^5C_1=5$ CC _ _ $^5C_2=10$ OOC _ $^5C_1=5$ OO _ _ $^5C_2=10$ C _ _ _ $^5C_3=10$ O _ _ _ $^5C_3=10$	B1	Correct outcome/value for 1 identified scenario. Accept unsimplified. WWW
		M1	Add 5 or 6 values of appropriate scenarios only, no additional incorrect scenarios, no repeated scenarios. Accept unsimplified. Condone use of permutations.
	[Total =] 50	A1	
		3	
(d)	Both Os in group with a C $^5C_2=10$ Both Os in group without a C $^5C_2\times{}^3C_2=30$ One O in a C group, one not $^5C_1\times{}^4C_2=30$ One O with each C $({}^5C_1\times{}^4C_1)\div2!=10$	B1	A correct scenario calculated accurately. Accept unsimplified.
		M1	Add 3 or 4 correct scenario values, no incorrect scenarios, accept repeated scenarios. Accept unsimplified.
	[Total =] 80	A1	
	Alternative method for question 6(d)		
	CCO O^^ ^^^ $=^5C_2=10$ CC^ O^^ O^^ $=^5C_1\times{}^4C_2=30$ CC^ OO^ ^^^ $=^5C_1\times{}^4C_1=20$	B1	A correct scenario calculated accurately. Accept unsimplified.
	Total ways of making three groups $\frac{{}^9C_6\times{}^6C_3}{2\times2\times3}=140$ 140 – (*their* 10+ *their* 30+ *their* 20)	M1	Total subtract 2 or 3 correct scenario values, no incorrect scenarios. Accept unsimplified.
	80	A1	
		3	

83. 9709_s22_ms_53 Q: 7

Question	Answer	Marks	Guidance
(a)	$^{12}C_5 \times {}^{7}C_4$ [$\times {}^{3}C_3$]	M1	$^{12}C_r \times q$, $r = 3, 4, 5$ q a positive integer > 1, no + or − .__
		M1	$^{12}C_s \times {}^{12-s}C_t$ [$\times {}^{12-s-t}C_u$] $s = 3, 4, 5$; $t = 3, 4, 5 \neq s$; $u = 3, 4, 5 \neq s, t$
	Alternative method for question 7(a)		
	$\frac{12!}{5! \times 3! \times 4!}$	M1	12! ÷ by a product of three factorials.
		M1	$\frac{n!}{5! \times 3! \times 4!}$
	[792 × 35 =] 27 720	A1	CAO
		3	

Question	Answer	Marks	Guidance
(b)	4! (Lizo) × 6! (Kenny) × 2! (Martin) × 2! (Nantes)	M1	Product involving at least 3 of 4!, 6!, 2!, 2!
	× 3! (orders of K, M and N)	M1	w × 3! , w integer > 1.
	414 720	A1	WWW CAO
		3	
(c)	$^{7}C_4$ (adults) × $^{4}C_1 \times {}^{3}C_1$	M1	$^{7}C_4 \times b$, b integer > 1 no + or – .
	420	A1	
		2	
(d)	K not L $^{5}C_3 \times {}^{8}C_3 = 560$ L not K $^{5}C_3 \times {}^{8}C_3 = 560$ L and K $^{5}C_2 \times {}^{8}C_3 = 560$	M1	$^{8}C_3$(or $^{8}P_3$) × c for one of the products or $^{5}C_3$ (or $^{5}P_3$)× c, positive integer >1 for first 2 products only.
		M1	Add 2 or 3 correct scenarios only values, no additional incorrect scenarios, no repeated scenarios. Accept unsimplified.
	[Total or Difference=] 1680	A1	
	Alternative method for question 7(d)		
	Total no of ways – neither L nor K Total = $^{7}C_4 \times {}^{8}C_3 = 1960$ Neither K nor L = $^{5}C_4 \times {}^{8}C_3 = 280$	M1	$^{8}C_3 \times c$, c a positive integer >1.
		M1	Subtracting the number of ways with neither from their total number of ways.
	[Total or Difference=] 1680	A1	

Question	Answer	Marks	Guidance
(d)	**Alternative method for question 7(d)**		
	Subtracting K and L from sum of K and L K $^{6}C_3 \times {}^{8}C_3 = 1120$ L $^{6}C_3 \times {}^{8}C_3 = 1120$ L and K $^{5}C_2 \times {}^{8}C_3 = 560$ 1120 + 1120 – 560 = 1680	M1	$^{8}C_3 \times c$, c a positive integer >1.
		M1	Subtracting number of ways with both from sum of number of ways with K and number of ways with L.
	[Total or Difference=] 1680	A1	
		3	

84. 9709_m21_ms_52 Q: 6

Question	Answer	Marks	Guidance
(a)	$\frac{11!}{2!2!2!}$	**M1**	11! alone as numerator. 2! × m! × n! on denominator, m = 1, 2, n = 1, 2. no additional terms, no additional operations.
	4989600	**A1**	Exact answer only.
		2	

Question	Answer	Marks	Guidance
(b)	**Method 1** R ^ ^ ^ ^ ^ ^ ^ R		
	Arrange the 7 letters CTEPILL = $\frac{7!}{2!}$ Number of ways of placing As in non-adjacent places = ${}^8C_2$ $\frac{7!}{2!}\times{}^8C_2$	**B1**	$\frac{7!}{2!}\times k$ seen, k an integer > 1.
		M1	$m\times n(n-1)$ or $m\times{}^nC_2$ or $m\times{}^nP_2$, n = 7, 8 or 9, m an integer > 1.
		M1	$\frac{7!}{p!}\times{}^8C_2$ or $\frac{7!}{p!}\times{}^8P_2$, p integer ⩾ 1, condone 2520×28.
	= 70560	**A1**	Exact answer only. **SC B1** 70560 from M0, M1 only.
	Method 2 [Arrangements Rs at ends – Arrangements Rs at ends and As together]		
	Total arrangements with R at beg. and end = $\frac{9!}{2!2!}$ Arrangements with R at ends and As together = $\frac{8!}{2!}$ With As not together = $\frac{9!}{2!2!}-\frac{8!}{2!}$	**M1**	$\frac{9!}{2!m!}-k$, 90720 > k integer > 1, m = 1, 2.
		B1	$s-\frac{8!}{2!}$, s an integer >1
		M1	$\frac{9!}{p}-\frac{8!}{q}$, p, q integers ⩾ 1, condone 90720 – 20160.
	[90720 – 20160] = 70560	**A1**	Exact answer only. **SC B1** 70560 from M0, M1 only.
		4	

Question	Answer	Marks	Guidance
(c)	**Method 1**		
	R R A L _ _ $^5C_2$ = 10 R R A L L _ $^5C_1$ = 5 R R A A L _ $^5C_1$ = 5 R R A A L L = 1	**M1**	^5C_x seen alone or $^5C_x \times k$, $2 \geqslant k \geqslant 1$, k an integer, $0 < x < 5$ linked to an appropriate scenario.
		A1	$^5C_2 \times k$, $k = 1$ oe or $^5C_1 \times m$, $m = 1,2$ oe alone. **SC** if ^5C_x not seen. **B2** for 5 or 10 linked to the appropriate scenario WWW.
		M1	Add outcomes from 3 or 4 identified correct scenarios only, accept unsimplified. $^2C_w \times {}^2C_x \times {}^2C_y \times {}^5C_z$, $w+x+y+z=6$ identifies w Rs, × As and y Ls.
	[Total =] 21	**A1**	WWW, only dependent on 2nd M mark. Note: $^5C_2 + {}^5C_1 + {}^5C_1 + 1 = 21$ is sufficient for 4/4.
			SC not all (or no) scenarios identified. **B1** 10 + 5 + 5 + 1 **DB1** = 21
	Method 2 – Fixing RRAL first. N.B. No other scenarios can be present anywhere in solution.		
	R R A L ^ ^ = $^7C_2$	**M1**	^7C_x seen alone or $^7C_x \times k$, $2 \geqslant k \geqslant 1$, k an integer, $0<x<7$. Condone ^7P_x or $^7P_x \times k$, $2 \geqslant k \geqslant 1$, k an integer, $0<x<7$.
		M1	$^7C_2 \times k$, $2 \geqslant k \geqslant 1$oe
		A1	$^7C_2 \times k$, $k = 1$oe no other terms.
	[Total =] 21	**A1**	Value stated.
		4	

85. 9709_s21_ms_51 Q: 1

Question	Answer	Marks	Guidance
	RRRRB $^8C_4 \times {}^4C_1 = 280$ BBBBR $^8C_1 \times {}^4C_4 = 8$ RRRRR $^8C_5 = 56$	M1	$^8C_x \times {}^4C_y$ with $x + y = 5$. x, y both integers, $1 \leqslant x \leqslant 5$, $0 \leqslant y \leqslant 4$ condone $^8C_1 \times 1$
		A1	Two correct outcomes evaluated
		M1	Add 2 or 3 identified correct scenarios only (no additional terms, not probabilities)
	[Total =] 344	A1	WWW, only dependent on 2nd M mark
		4	**SC** not all (or no) scenarios identified **B1** 280 + 8 + 56 **DB1** 344

86. 9709_s21_ms_53 Q: 6

Question	Answer	Marks	Guidance
(a)	$\frac{11!}{2!3!}$	M1	11! alone on numerator – must be a fraction. $k! \times m!$ on denominator, $k = 1, 2$, $m = 1, 3, 1$ can be implied but cannot both = 1. No additional terms
	3326400	A1	Exact value only
		2	
(b)	8! = 40320	B1	Evaluate, exact value only
		1	
(c)	$\frac{9!}{3!}\times 7$	M1	$\frac{9!}{3!}\times k$ seen, k an integer > 0, no +, – or ÷
		M1	7 × an integer seen in final answer, no +, – or ÷
	423360	A1	Exact value only
	Alternative method for Question 6(c)		
	${}^9C_3 \times 7! \left(\times \frac{3!}{3!}\right)$	M1	$9C3\times k$ seen, k an integer > 0, no + or –
		M1	$7! \times k$ seen, , k an integer > 0, no + or –
	423360	A1	Exact value only but there must be evidence of $\times\frac{3!}{3!}$

Question	Answer	Marks	Guidance
(c) cont'd	**Alternative method for Question 6(c)**		
	$3\times 7\times\frac{8!}{2!}$	M1	$3\times\frac{8!}{2!}\times k$ seen, k an integer > 0, no + or –
		M1	7 × an integer seen in final answer, no +, – or ÷
	423360	A1	Exact value only
	Alternative method for Question 6(c)		
	$7\times\frac{2}{11}\times\frac{9}{10}\times\frac{8}{9}\times\frac{7}{8}\times\frac{1}{7}\times$ total no. of arrangements	M1	Product of correct five fractions × k seen, k an integer > 0, no + or –
		M1	7×'total no of arrangements' ×k seen, k an integer > 0, no + or –
	423360	A1	Exact value only
	Alternative method for Question 6(c)		
	No E between the Rs $\quad \frac{{}^6C_3\times 3!\times 7!}{3!} = 100800$ 1E between the Rs $\quad \frac{{}^6C_2\times 3!\times 7!}{2!} = 226800$ 2Es between the Rs $\quad {}^6C_1\times 3!\times 7! = 90720$ 3Es between the Rs $\quad 7! = 5040$	M1	Finding the correct number of ways for no, 1 or 2 Es between the Rs, accept unsimplified.
		M1	Adding the number of ways for 3 or 4 correct scenarios
	$\left[\text{Total} = 7!\times(20+45+18+1) = 7!\times 84 =\right] 423360$	A1	CAO
		3	

Question	Answer	Marks	Guidance
(d)	E E R _ _ $^6C_2 = 15$ E E R R _ $^6C_1 = 6$ E E E R _ $^6C_1 = 6$ E E E R R $^6C_0 = 1$	M1	Identifying four correct scenarios only.
		B1	Correct number of selections unsimplified for 2 or more scenario.
		M1	Adding the number of selections for 3 or 4 identified correct scenarios only, accept unsimplified. $^3C_x \times {}^2C_y \times {}^6C_z$, $x+y+z=5$ correctly identifies x Es and y Rs
	[Total =] 28	A1	WWW, only dependent upon 2nd M mark.
	Alternative method for Question 6(d) – Fixing EER first. No other scenarios can be present anywhere in solution.		
	E E R ^ ^ = $^8C_2$	M1	^8C_x seen alone or $^8C_x \times k$, , $k = 1$ or 2, $0<x<8$ Condone ^8P_x or $^8P_x \times k$, $k = 1$ or 2, $0<x<8$
		B1	$^8C_2 \times k$, $k = 1$ or 2 OE
		M1	$^8C_2 \times k$, $k = 1$ OE and no other terms
	[Total =] 28	A1	Value stated
		4	

87. 9709_w21_ms_52 Q: 2

Question	Answer	Marks	Guidance
(a)	$^{11}C_5 \times {}^4C_1$	M1	$^{11}C_5 \times {}^4C_1$ condone $^{11}P_5 \times {}^4P_1$ no +, –, × or ÷.
	1848	A1	CAO as exact.
		2	
(b)	**Method 1** [Identifying scenarios]		
	[Neither selected =] $^{13}C_6$ [= 1716] [Only Jane selected =] $^{13}C_5$ [= 1287] [Only Kate selected =] $^{13}C_5$ [= 1287]	M1	Either $^{13}C_6$ seen alone or $^{13}C_5$ seen alone or × 2 (condone $^{13}P_n$, $n = 5,6$).
	[Total =] 1716 + 1287 + 1287	M1	Three correct scenarios only added, accept unsimplified (values may be incorrect).
	4290	A1	
	Method 2 [Total number of selections – selections with Jane and Kate both picked]		
	$^{15}C_6$ - $^{13}C_4$ [= 5005 – 715]	M1	$^{15}C_6 - k$, k a positive integer < 5005, condone $^{15}P_6$.
		M1	$m - {}^{13}C_4$, m integer > 715, condone $n - {}^{13}P_4$, $n > 17\,160$.
	4290	A1	
		3	
			SC Where the condition of **2(a)** is also applied in **2(b)**, the final answer is 1512 **SC M1 M1 A0** max. The method marks can be earned for the equivalent stages in each method. Method 1 $^4C_1 \times {}^9C_5 + {}^4C_1 \times {}^9C_4 \times 2$ Method 2 $^4C_1 \times {}^{11}C_5 - {}^4C_1 \times {}^9C_3$

88. 9709_w21_ms_52 Q: 4

Question	Answer	Marks	Guidance
(a)	$\frac{9!}{3!}$	M1	$\frac{9!}{e!}$, $e = 2, 3$
	60 480	A1	
		2	

Question	Answer	Marks	Guidance
(b)	$\frac{7!}{3!}\times 2\times 6$	M1	$\frac{7!}{3!}\times k$ seen, k an integer > 0.
		M1	$\frac{m!}{n!}\times 2\times q$ $7 \leqslant m \leqslant 9, 1 \leqslant n \leqslant 3, 1 \leqslant q \leqslant 8$ all integers.
		M1	$\frac{m!}{n!}\times p\times 6$ $7 \leqslant m \leqslant 9, 1 \leqslant n \leqslant 3, 1 \leqslant p \leqslant 2$ all integers. (Accept 3P2 for 6) If **M0 M0 M0** awarded, **SC M1** for $t\times 12$, t an integer $\geqslant 20$, $\frac{5!}{3!}$.
	10 080	A1	Exact value.
	Alternative method for question 4(b)		
	$\frac{{}^{7}\mathrm{P}_{2}\times 6!\times 2}{3!}$	M1	$\frac{6!}{3!}\times k$ seen, k an integer > 0.
		M1	$\frac{m!}{n!}\times {}^{7}\mathrm{P}_{2}\times q$ $m = 6,9, 1 \leqslant n \leqslant 3, 1 \leqslant q \leqslant 2$ all integers.
		M1	$\frac{m!}{n!}\times {}^{7}\mathrm{P}_{r}\times 2$ $m = 6, 9, 1 \leqslant n \leqslant 3, 1 \leqslant r \leqslant 5$ all integers. If **M0 M0 M0** awarded, **SC M1** for $t\times 84$, t an integer $\geqslant 20$, $\frac{5!}{3!}$.
	10 080	A1	Exact value.

Question	Answer	Marks	Guidance
(b)	**Alternative method for question 4(b)**		
	$\frac{7!}{3!}\times$4P2	M1	$\frac{7!}{3!}\times k$ seen, k an integer > 0.
		M1	$t\times {}^{4}\mathrm{P}_{2}$ or 12, t an integer $\geqslant 20$, $\frac{5!}{3!}$.
		M1	$\frac{m!}{n!}\times$4P2 $7 \leqslant m \leqslant 9, 1 \leqslant n \leqslant 3$ all integers.
	10 008	A1	Exact value.
		4	

89. 9709_w21_ms_53 Q: 1

Question	Answer	Marks	Guidance
	${}^{23}\mathrm{C}_{17}$	M1	${}^{23}\mathrm{C}_{x}$ or ${}^{y}\mathrm{C}_{17}$ or ${}^{z}\mathrm{C}_{6}$, x, y or z are integers no +, –, × or ÷.
	100947	A1	CAO
		2	

90. 9709_m20_ms_52 Q: 1

Question	Answer	Marks	Guidance
	${}^{38}\mathrm{C}_{r}$ or ${}^{n}\mathrm{C}_{34}$	M1	Either expression seen OE, no other terms, condone x1
	${}^{38}\mathrm{C}_{34}$	A1	Correct unsimplified OE
	73815	A1	If M0, **SCB1** ${}^{38}\mathrm{C}_{34}$ x k, k an integer
		3	

91. 9709_m20_ms_52 Q: 4

Question	Answer	Marks	Guidance
(a)	R ^ ^ ^ ^ ^ ^ ^ ^ ^ ^ R $\frac{9!}{3!6!}$	M1	9! Alone on numerator, 3! × k or 6! × k on denominator
	= 84	A1	
		2	
(b)	^ (B B B) ^ ^ ^ ^ ^	M1	$\frac{7!}{6!}\times k$ or $7k$ seen, k an integer > 0
	$\frac{7!}{6!}\times\frac{8\times7}{2}$	M1	$m\times n(n-1)$ or $m\times {}^nC_2$ or $m\times {}^nP_2$, n=7, 8 or 9, m an integer > 0
		M1	n = 8 used in above expression
	= 196	A1	
	Alternative for question 4(b)		
	[Arrangements, blues together – Arrangements with blues together and reds together =] $\frac{9!}{2!6!}-\frac{8!}{6!}$	M1	9! Seen alone or as numerator with subtraction
	= [252 – 56]	M1	8! Seen alone or as numerator in a second term and no other terms
		M1	All terms divided by 6! x k, k an integer
	= 196	A1	
		4	

92. 9709_s20_ms_51 Q: 2

Question	Answer	Marks
(a)	6!	M1
	720	A1
		2
(b)	Total number: $\frac{9!}{3!2!}(30240)$	M1
	Number with Ls together = $\frac{8!}{3!}(6720)$	M1
	Number with Ls not together = $\frac{9!}{3!2!}-\frac{8!}{3!}$ = 30 240 – 6720	M1
	23 520	A1
	Alternative method for question 2(b)	
	$\frac{7!}{3!}\times\frac{8\times7}{2}$	
	7! × k in numerator, k integer ≥ 1	M1
	8 × 7 × m in numerator or 8C2 × m, m integer ≥ 1	M1
	3! in denominator	M1
	23 520	A1
		4

93. 9709_s20_ms_51 Q: 4

Question	Answer	Marks
	Scenarios: 2P 3V 2G $^8C_2 \times {}^4C_2 \times {}^6C_3 = 28 \times 6 \times 20 = 3360$ 2P 4V 1G $^8C_2 \times {}^4C_1 \times {}^6C_4 = 28 \times 4 \times 15 = 1680$ 3P 3V 1G $^8C_3 \times {}^4C_1 \times {}^6C_3 = 56 \times 4 \times 20 = 4480$ 4P 2V 1G $^8C_4 \times {}^4C_1 \times {}^6C_2 = 70 \times 4 \times 15 = 4200$ (**M1** for $^8C_r \times {}^4C_r \times {}^6C_r$ with $\sum r = 7$)	**M1**
	Two unsimplified products correct	**B1**
	Summing the number of ways for 3 or 4 correct scenarios	**M1**
	Total: 13 720	**A1**
		4

94. 9709_s20_ms_52 Q: 6

Question	*Answer*	Marks
(a)	$\frac{8!}{3!}$	**M1**
	6720	**A1**
		2

Question	Answer	Marks
(b)	Total number = $\frac{10!}{2!3!}$ (302400) (A)	**B1**
	With Es together = $\frac{9!}{3!}$ (60480) (B)	**B1**
	Es not together = *their* (A) – *their* (B)	**M1**
	241920	**A1**
	Alternative method for question 6(b)	
	_ ^ _ ^ _ ^ _ ^ _ ^ _ ^ _ ^ _ $\frac{8!}{3!} \times \frac{9 \times 8}{2}$	
	8! × *k* in numerator, *k* integer ≥ 1, denominator ≥ 1	**B1**
	3! × *m* in denominator, *m* integer ≥ 1	**B1**
	Their $\frac{8!}{3!}$ Multiplied by $^9C_2$ (OE) only (no additional terms)	**M1**
	241920	**A1**
		4

Question	Answer	Marks
(c)	Scenarios: E M M M $^5C_0 = 1$ E M M _ $^5C_1 = 5$ E M _ _ $^5C_2 = 10$	**M1**
	Summing the number of ways for 2 or 3 correct scenarios	**M1**
	Total = 16	**A1**
		3

95. 9709_w20_ms_53 Q: 3

Question	Answer	Marks	Guidance
(a)	Scenarios: 6W 0M $^9C_6 = 84$ 5W 1M $^9C_5 \times {}^5C_1 = 126 \times 5 = 630$ 4W 2M $^9C_4 \times {}^5C_2 = 126 \times 10 = 1260$	M1	Correct number of ways for either 5 or 4 women, accept unsimplified
		M1	Summing the number of ways for 2 or 3 correct scenarios (can be unsimplified), no incorrect scenarios.
	Total = 1974	A1	
		3	
(b)	Total number of ways = $^{14}C_6$ (3003) Number with sister and brother = $^{12}C_4$ (495) Number required = $^{14}C_6 -$	M1	$^{14}C_6$ – a value
	$^{12}C_4 = 3003 - 495$	M1	$^{12}C_x$ or $^nC_4$ seen on its own or subtracted from *their* total, $x \leqslant 6$, $n \leqslant 13$
	2508	A1	
	Alternative method for question 3(b)		
	Number of ways with neither = $^{12}C_6 = 924$	M1	$^{12}C_6$ + a value
	Number of ways with either brother or sister (not both) = $^{12}C_5 \times 2$ (= 792 × 2) = 1584	M1	$^{12}C_x \times 2$ or $^nC_5 \times 2$ seen on its own or added to *their* number of ways with neither, $x \leqslant 5$, $n \leqslant 12$
	Number required = 924 + 1584 = 2508	A1	
		3	

96. 9709_m19_ms_62 Q: 7

Question	Answer	Marks	Guidance
(i)	$\frac{9!}{2!3!}$	M1	9! alone on numerator, 2! and/or 3! on denominator
	= 30240	A1	Exact value, final answer
		2	
(ii)	A ^ ^ ^ A ^ ^ ^ A Arrangements = $\frac{6!}{2!} = 360$	B1	Final answer
		1	
(iii)	M ^ M ^ ^ ^ ^ ^ ^ = $\frac{7!}{3!} \times 7$	M1	7! in numerator, (considering letters not M)
		M1	Division by 3! only (removing repeated As)
		M1	Multiply by 7 (positions of M-M)
	= 5880	A1	Exact value, final answer
	Method 2 (choosing letter between Ms)		
	$1 \times \frac{6!}{2!} \times 7 + 4 \times \frac{6!}{3!} \times 7$	M1	6! in sum of 2 expressions $a6! + b6!$
		M1	Multiply by 7 in both expressions (positions of M-M)
	= 2520 + 3360	M1	$\frac{c}{2!} + \frac{d}{3!}$ seen (removing repeated As)
	= 5880	A1	Exact value

Question	Answer	Marks	Guidance
(iii)	**Method 3**		
	(MAM) ^ ^ ^ ^ ^ ^ = 7!/2! = 2520	**M1**	7! in numerator (considering 6 letters + block)
	(MA'M) ^ ^ ^ ^ ^ ^ = 7!/3! × 4 = 840 × 4 = 3360	**M1**	Division by 2! and 3! seen in different terms
	Total = 2520 + 3360	**M1**	Summing 5 correct scenarios only
	= 5880	**A1**	Exact value
		4	
(iv)	M A ^ = $^4C_1$ = 4	**B1**	Final answer
		1	
(v)	M ^ ^ : $^4C_2$ = 6 M M ^ : $^4C_1$ = 4	**M1**	Either option M M ^ or M ^ ^ correct, accept unsimplified
	M M A : = 1 M A A : = 1 (M A _ : $^4C_1$ = 4)	**M1**	Add 4 or 5 correct scenarios only
	Total = 16	**A1**	Value must be clearly stated
	Method 2		
	M M ^ = $^5C_1$ = 5	**M1**	Either option M M ^ or M ^ ^ correct, accept unsimplified
	M ^ ^ = $^5C_2$ = 10	**M1**	Adding 2 or 3 correct scenarios only
	M A A = = 1 Total = 16	**A1**	Value must be clearly stated
		3	

97. 9709_s19_ms_61 Q: 8

Question	Answer	Marks	Guidance
(i)	($^9C_4$ =) 126	**B1**	
		1	
(ii)	$^7C_2$	**B1**	^7C_x or $^yC_2$ (implied by correct answer) or ^7P_x or ^7P_y, seen alone
	= 21	**B1**	correct answer
		2	

Question	Answer	Marks	Guidance
(iii)	_ C_1 (B_1 B_2 B_3) C_2 _ C_3 _ C_4 _ C_5 _ C_6	**B1**	3! or 6! seen alone or multiplied by k > 1 need not be an integer
	3! × 6! × 7	**B1**	3! and 6! seen multiplied by k > 1, integer, no division
	= 30240	**B1**	Exact value
	Alternative method for question 8(iii)		
	C_1 (B_1 B_2 B_3) C_2 C_3 C_4 C_5 C_6	**B1**	3! or 7! seen alone or multiplied by k > 1 need not be an integer
	3! × 7!	**B1**	3! and 7! seen multiplied by k > or = 1, no division
	= 30240	**B1**	Exact value
		3	
(iv)	C_1 _ C_2 _ C_3 _ C_4 _ C_5 _C_6	**B1**	6! or 4! X 6P2 seen alone or multiplied by k > 1, no division (arrangements of cars)
	6! × 5P3 *or* 6! × 5 × 4 × 3 or 6! x 3! x10	**B1**	Multiply by 5P3 oe i.e. putting Bs in between 4 of the Cs OR multiply by 3! x n where n = 7, 8, 9, 10 (number of options)
	= 43200	**B1**	Correct answer
		3	

98. 9709_s19_ms_62 Q: 7

Question	Answer	Marks	Guidance
(a)	${}^6C_3 \times {}^3C_2 \times {}^1C_1$	M1	${}^6C_a \times {}^{6-a}C_b \times {}^{6-a-b}C_{6-a-b}$ seen oe ${}^{6-a-b}C_{6-a-b}$ can be implied by 1 or omission, condone use of permutations,
	$= 20 \times 3$	A1	Any correct method seen no addition/additional scenarios
	$= 60$	A1	Correct answer
	Alternative method for question 7(a)		
	$\frac{{}^6P_6}{{}^3P_3 \times {}^2P_2 \times {}^1P_1} = \frac{6!}{3! \times 2!}$	M1	${}^6P_6$ / (${}^nP_n$ x k) with $3 \geqslant n > 1$and $6 \geqslant$ k an integer $\geqslant 1$, not 6!/1
		A1	Correct method with no additional terms
	$= 60$	A1	Correct answer
		3	
(b)(i)	$\frac{4!}{3!} \times \frac{3!}{2!} \times 2$	M1	A single expression with either $4!/3! \times k$ or $3!/2! \times$ k, k a positive integer seen oe (condone 2 identical expressions being added)
		M1	Correctly multiplying *their* single expression by 2 or 2 identical expressions being added.
	$= 24$	A1	Correct answer
		3	

Question	Answer	Marks	Guidance
(b)(ii)	Total no of arrangements = $\frac{7!}{2!3!} = 420$ (A)	B1	Accept unsimplified
	No with 2s together = $\frac{6!}{3!} = 120$ (B)	B1	Accept unsimplified
	With 2s not together: *their* (A) – *their* (B)	M1	Subtraction indicated, possibly by *their* answer, no additional terms present
	= 300 ways	A1	Exact value www
	Alternative method for question 7(b)(ii)		
	3 _ 7 _ 7 _ 7 _ 8 _		
	$\frac{5!}{3!} \times \frac{6 \times 5}{2}$	B1	k x 5! in numerator, k a positive integer
		B1	m x $3!$ In denominator, m a positive integer
		M1	*Their* 5!/3! multiplied by ${}^6C_2$ only (no additional terms)
	= 300 ways	A1	Exact value www
		4	

99. 9709_s19_ms_63 Q: 3

Question	Answer	Marks	Guidance
(i)	$9! \times 2$	B1	9! seen multiplied by $k \geqslant 1$, no addition
	= 725760	B1	Exact value
		2	
(ii)	Eg $(K_1K_2K_3K_4K_5)$ A A A (U_1U_2) A	B1	2! or 5! seen mult by k > 1, no addition (arranging Us or Ks)
	$= 5! \times 2! \times 6!$	B1	6! Seen mult by k > 1, no addition (arranging AAAAKU)
	= 172800	B1	Exact value
		3	

100. 9709_s19_ms_63 Q: 4

Question	Answer	Marks	Guidance
(i)	M(8) W(4) 4 2 in ${}^8C_4 \times {}^4C_2 = 420$ ways 5 1 in ${}^8C_5 \times {}^4C_1 = 224$ ways 6 0 in ${}^8C_6 \times {}^4C_0 = 28$ ways	**B1**	One unsimplified product correct
		M1	Summing the number of ways for 2 or 3 correct scenarios (can be unsimplified), no incorrect scenarios
	Total 672 ways	**A1**	Correct answer
		3	

Question	Answer	Marks	Guidance
(ii)	Total number of selections = ${}^{12}C_6 = 924$ (A)	**M1**	${}^{12}C_x$ – (subtraction seen), accept unsimplified
	Selections with males together = ${}^{10}C_4 = 210$ (B)	**A1**	Correct unsimplified expression
	Total = (A) – (B) = 714	**A1**	Correct answer
	Alternative method for question 4(ii)		
	No males + Only male 1 + Only male 2 = ${}^{10}C_6 + {}^{10}C_5 + {}^{10}C_5$	**M1**	${}^{10}C_x + 2 \times {}^{10}C_y$, $x \neq y$ seen, accept unsimplified
	= 210 + 252 + 252	**A1**	Correct unsimplified expression
	= 714	**A1**	Correct answer
	Alternative method for question 4(ii)		
	Pool without male 1 + Pool without male 2 – Pool without either male	**M1**	$2 \times {}^{11}C_x - {}^{10}C_x$
	= ${}^{11}C_6 + {}^{11}C_6 - {}^{10}C_6$ = 462 + 462 – 210	**A1**	Correct unsimplified expression
	= 714	**A1**	Correct answer
		3	

101. 9709_w19_ms_61 Q: 6

Question	Answer	Marks	Guidance
(i)	$\frac{9!}{2!} = 181\,440$	B1	Exact value
		1	
(ii)	Total no of ways = $\frac{12!}{2!4!} = 9\,979\,200$ (A)	B1	Accept unevaluated
	With Ss together = $\frac{11!}{4!} = 1\,663\,200$ (B)	B1	Accept unevaluated
	With Ss not together = (B) – (A)	M1	Correct or $\frac{12!}{m} - \frac{8!}{n}, m, n$ integers > 1 or *their* identified total – *their* identified Ss together
	8 316 000	A1	Exact value
	Alternative method for question 6(ii)		
	_ T _ E _ E _ P _ L _ E _ C _ H _ A _ E _	B1	$10! \times k$ in numerator k integer $\geqslant 1$
	$\frac{10!}{4!} \times \frac{11 \times 10}{2!}$	B1	$4! \times k$ in numerator k integer $\geqslant 1$
	$\frac{\textit{their}\,10!}{\textit{their}\,4!} \times {}^{11}C_2$ or ${}^{11}P_2$	M1	OE
	8 316 000	A1	Exact value
		4	

Question	Answer	Marks	Guidance
(iii)	S E E E : 1	M1	${}^{6}C_x$ seen alone or times $K > 1$
	S E E _ : ${}^{6}C_1 = 6$ S E _ _ : ${}^{6}C_2 = 15$ S _ _ _ : ${}^{6}C_3 = 20$	B1	${}^{6}C_3$ or ${}^{6}C_2$ or ${}^{6}C_1$ alone
	Add 3 or 4 correct scenarios	M1	No extras
	Total = 42	A1	
		4	

102. 9709_w19_ms_63 Q: 2

Question	Answer	Marks	Guidance
(i)	$\frac{9!}{2!3!} = 30240$	B1	9! Divided by at least one of 2! or 3!
		B1	Exact value
		2	
(ii)	D _ _ _ _ _ _ _ R: $\frac{7!}{2!2!} = 1260$ D _ _ _ _ _ _ _ O: $\frac{7!}{3!} = 840$	B1	7! Seen alone or as numerator in a term, can be multiplied not + or –
		B1	One term correct, unsimplified
	Total = 2100	B1	Final answer
		3	

103. 9709_w19_ms_63 Q: 3

Question	Answer	Marks	Guidance
(i)	3*A* 2*D* 2*M* : ${}^6C_3 \times {}^5C_2 \times {}^4C_2$ (= 1200) 4*A* 2*D* 1*M* : ${}^6C_4 \times {}^5C_2 \times {}^4C_1$ (= 600) 3*A* 3*D* 1*M* : ${}^6C_3 \times {}^5C_3 \times {}^4C_1$ (= 800)	**M1**	${}^6C_x \times {}^5C_y \times {}^4C_z$, $x + y + z = 7$
		A1	2 correct products, allow unsimplified
		M1	Summing their totals for 3 correct scenarios only
	Total = 2600	**A1**	Correct answer **SC1** ${}^6C_3 \times {}^5C_2 \times {}^4C_1 \times {}^9C_1 = 7200$
		4	

Question	Answer	Marks	Guidance
(ii)	${}^7C_4 \times 1$	**B1**	${}^7C_3$ or ${}^7C_4$ seen anywhere
	35	**B1**	
		2	

104. 9709_m18_ms_62 Q: 2

Question	Answer	Marks	Guidance
(i)	1 L: ${}^6C_2 = 15$	**B1**	
		1	
(ii)	No L: ${}^6C_3 = 20$ (1 L: ${}^6C_2 = 15$)	**M1**	Either 0L or 2L correct unsimplified
	2 L: ${}^6C_1 = 6$	**M1**	Summing the 3 correct scenarios
	Total = 41	**A1**	
		3	

105. 9709_m18_ms_62 Q: 6

Question	Answer	Marks	Guidance
(i)	$3! \times \frac{4!}{3!} \times 2$	**M1**	3! oe seen multiplied by integer $\geqslant 1$, no addition
		M1	4!/3! oe seen multiplied by integer > 1, no addition
	= 48	**A1**	
		3	
(ii)	*EITHER:* Even = Total number of arrangements – Odd numbers $= 7!/3! - 3\times\frac{6\times5\times4\times3\times2\times1}{3!} = (7!/3! - 6!/2!)$ = 840 −360	**B1** **B1**	7!/3! – 6!/2! OE
	= 480	**B1**	
	OR: No of arrangements ending in 8: $\frac{6!}{3!}$	**B1**	No. ending in 8 or no. ending in 6 correct unsimplified
	No ending in 6: 6!/2!	**B1**	Both correct and added unsimplified
	Total: $\frac{6!}{3!} + 6!/2 = 120 + 360 = 480$	**B1**	
		3	

106. 9709_s18_ms_61 Q: 7

Question	Answer	Marks	Guidance
(i)	$\frac{9!}{2!2!} = 90720$	B1	Must see 90720
		1	
(ii)	**Method 1** ↑ * * * * * A	B1	5! seen multiplied (arrangement of consonants allowing repeats)
	No. arrangements of consonants × ways of inserting vowels =	B1	${}^6P_4$ oe (i.e. $6 \times 5 \times 4 \times 3$, ${}^6C_4 \times 4!$) seen mult (allowing repeats) no extra terms
	$\frac{5!}{2!} \times \frac{{}^6P_4}{2!}$	B1	Dividing by at least one 2! (removing at least one set of repeats)
	Answer $\frac{{}^6P_4}{2!} \times \frac{5}{2} = 10\,800$	B1	Correct final answer
		4	
(iii)	${}^5C_3 = 10$	M1	${}^5C_x$ or ${}^5P_x$ seen alone, $x = 2$ or 3
		A1	Correct final answer not from ${}^5C_2$
		2	

Question	Answer	Marks	Guidance
(iv)	**Method 1** Considering separate groups	M1	Considering two scenarios of MME or EEM or MMEE with attempt, may be probs or perms
	MME** = ${}^5C_2 = 10$ MEE** = ${}^5C_2 = 10$ MMEE* = ${}^5C_1 = 5$	M1	Summing three appropriate scenarios from the four need ${}^5C_x$ seen in all of them
	ME*** = ${}^5C_3 = 10$ see **(iii)** Total = 35	A1	Correct final answer
	Method 2 Considering criteria are met if ME are chosen	M1	${}^7C_x$ only seen, no other terms
		M1	${}^xC_3$ only seen, no other terms
	ME *** = ${}^7C_3 = 35$	A1	Correct final answer
		3	

107. 9709_s18_ms_62 Q: 6

Question	Answer	Marks	Guidance
(a)(i)	(AAAIU) * * * * Arrangements of vowels/repeats × arrangements of (consonants & vowel group) =	M1	$k \times 5!$ (k is an integer, $k \geqslant 1$)
	$\frac{5! \times 5!}{3!}$	M1	$\frac{m}{3}!$ (m is an integer, $m \geqslant 1$) **Both** Ms can **only** be awarded if expression is **fully correct**
	= 2400	A1	Correct answer
		3	
(a)(ii)	E.g. R * * * T * * * L . Arrangements of consonants RL, RS, SL = ${}^3P_2 = 6$ Arrangements of remaining letters = $\frac{6!}{3!} = 120$	M1	$k \times \frac{6!}{3!}$ **or** $k \times {}^3P_2$ **or** $k \times {}^3C_2$ **or** $k \times 3!$ **or** $k \times 3 \times 2$ (k is an integer, $k \geqslant 1$), no irrelevant addition
	Total 120×6	M1	Correct unsimplified expression or $\frac{6!}{3!} \times {}^3C_2$
	= 720 ways	A1	Correct answer
		3	

Question	Answer	Marks	Guidance
(b)	**Method 1** N(2) R(8) Br(4) 1 2 1 $= 2\times{}^8C_2\times4 = 224$	**M1**	Multiply 3 combinations, ${}^2C_x\times{}^8C_y\times{}^4C_z$. Accept ${}^2C_1 = 2$ etc.
	2 1 1 $= 1\times{}^8C_1\times4 = 32$ 1 1 2 $= 2\times8\times{}^4C_2 = 96$	**A1**	3 or more options correct unsimplified
	2 0 2 $= 1\times1\times{}^4C_2 = 6$ 1 0 3 $= 2\times1\times4 = 8$	**M1**	Summing *their* values of 4 or 5 legitimate scenarios (no extra scenarios)
	Total = 366 ways	**A1**	Correct answer
	Method 2 ${}^{14}C_4$ – (2N2R or 1N3R or 4R or 3R1B or 2R2B or 1R3B or 4B)	**M1**	'${}^{14}C_4 - k$' seen, k an integer from an expression containing ${}^8C_x$
	$1001 - (1\times{}^8C_2 + 2\times{}^8C_3 + {}^8C_4 + {}^8C_3\times4 + {}^8C_2\times{}^4C_2 + 8\times4 + 1)$	**A1**	4 or more 'subtraction' options correct unsimplified, may be in a list
	$1001 - (28 + 112 + 70 + 224 + 168 + 32 + 1)$	**M1**	*Their* ${}^{14}C_4$ – [*their* values of 6 or more legitimate scenarios] (no extra scenarios, condone omission of final bracket)
	= 366	**A1**	Correct answer
		4	

108. 9709_s18_ms_63 Q: 7

Question	Answer	Marks	Guidance
(i)	****E**** Other letters arranged in $\frac{8!}{2!3!}$ = 3360 ways	**M1**	Mult by 8! or ${}^8P_8$ oe (arrangements ignoring repeats)
		A1	Correct final answer www
	OR $\frac{8\times7\times6\times5\times4\times4\times3\times2\times1}{4!2!} = 3360$ ways	**M1**	Correct numerator (161 280)
		A1	Correct final answer www
	Total:	**2**	
(ii)	* * * * * ↑ Arrangements other letters × ways Es inserted $= \frac{5!}{2!}\times{}^6C_4\left(\frac{5!}{2!}\times\frac{{}^6P_4}{4!}\right)$	**M1**	k mult by ${}^6C_4$ or ${}^6P_4$ oe (ways to insert Es ignoring repeats), k can = 1 or k mult by $\frac{5!}{2!}$
		M1	Correct unsimplified expression or $\frac{5!}{2!}\times{}^6P_4$
	= 900 ways	**A1**	Correct answer
	OR Total no of ways – no of ways with Es touching 9!/(4! × 2!) – … or 7 560 – … $\frac{6!}{2!} + {}^6P_2\times\frac{5!}{2!} + \frac{{}^6P_2}{2!}\times\frac{5!}{2!} + \frac{{}^6P_3}{2!\times\frac{5!}{2!}}$ = 360 + 1800 + 900 + 3600 = 6660	**M1**	7560 unsimplified – k
		M1	Attempting to find four ways of Es touching (4 Es, 3Es and a single, 2 lots of 2 Es, 2 Es and 2 singles)
	7 560 – 6 660 = 900	**A1**	Correct answer

Question	Answer	Marks	Guidance
(ii)	OR Adding the number of ways with the first E in the 1st (E_1), 2nd (E_2) or 3rd (E_3) position. $\frac{5!}{2!}(E_1 + E_2 + E_3)$ where $E_1 = 10$, $E_2 = 4$, $E_3 = 1$ $\frac{5!}{2!}(E_1 + E_2 + E_3)$	**M1**	For any values for E_1, E_2 and E_3
		M1	For any two correct values of E_1, E_2 and E_3
	600 + 240 + 60 = 900	**A1**	Correct answer
	Total:	**3**	
(iii)	EENN* in 3 ways	**B1**	Numerical value must be stated
	Total:	**1**	

Question	Answer	Marks	Guidance
(iv)	EE *** with no N: 1 way EEN** 3C2 or listing 3 ways EENN* 3 ways from **(iii)**	**M1**	Identifying the three different scenarios of EE, EEE or EEEE
		A1	Total no of ways with two Es (7 or 3 + 3 + 1)
	EEE** with no N: 3 ways EEEN* 3 ways EEENN 1 way	**A1**	Total no. of ways with 3 Es (7)
	EEEE* no N 3 ways EEEEN 1 way Total 18 ways	**A1**	Correct answer stated
	Method List containing ways with 2Es, 3Es and 4Es List containing at least 8 correct different ways List of all 18 correct ways Total 18	**M1**	At least 1 option listed for each of EE^^^, EEE^^, EEEE^
		A1	Ignore repeated options
		A1	Ignore repeated/incorrect options
		A1	Correct answer stated
	Total:	**4**	

109. 9709_w18_ms_61 Q: 1

Question	Answer	Marks	Guidance
	$^{9}C_{4} \times {}^{5}C_{3} \times {}^{2}C_{2}$	**B1**	$^{9}C_{4}$ or $^{9}C_{3}$ or $^{9}C_{2}$ seen *(1st group)*
	$=126 \times 10 \times 1$	**B1**	$^{5\text{ or }7}C_{3}$ or $^{6\text{ or }7}C_{4}$ or $^{6\text{ or }5}C_{2}$ times an integer *(2nd group)*
	$=1260$	**B1**	Correct answer
		3	

110. 9709_w18_ms_61 Q: 3

Question	Answer	Marks	Guidance
(i)	Scenarios are: 4V + 1C + 1DB: $^{11}C_{4} \times {}^{5}C_{1} \times {}^{4}C_{1}$	**M1**	$^{11}C_{a} \times {}^{5}C_{b} \times {}^{4}C_{c}$, $a+b+c=6$,
	4V + 2C: $^{11}C_{4} \times {}^{5}C_{2}$ 5V + 1C: $^{11}C_{5} \times {}^{5}C_{1}$	**B1**	2 correct unsimplified options
	6600 + 3300 + 2310	**M1**	Add 2 or 3 correct scenarios only
	= 12210	**A1**	Correct answer
		4	
(ii)	4! × 3!	**M1**	k multiplied by 3! or 4!, k an integer $\geqslant 1$
		A1	Correct unsimplified expression
	= 144	**A1**	Correct answer
		3	

111. 9709_w18_ms_62 Q: 4

Question	Answer	Marks	Guidance
(i)	5! × 6! ×2	**B1**	k×5! or m×6! (k,m integer, k,$m \geqslant 1$), no inappropriate addition
		B1	$n \times 5! \times 6!$ (n integer, $n \geqslant 1$), no inappropriate addition
	= 172800	**B1**	Correct final answer, isw rounding (www scores B3) All marks based on their **final** answer
		3	

Question	Answer	Marks	Guidance
(ii)	… G … G … G … G … G … No. ways girls placed × No. ways boys placed in gaps =	**M1**	$k \times 6!$ or $k \times {}^{7}P_{5}$ (k is an integer, $k \geqslant 1$) no inappropriate add. (${}^{7}P_{5} \equiv 7 \times 6 \times 5 \times 4 \times 3$ or ${}^{7}C_{5} \times 5!$)
	$6! \times {}^{7}P_{5}$	**M1**	Correct unsimplified expression
	= 1814400	**A1**	Correct exact final answer (ignore subsequent rounding)
		3	

PP' = $\frac{2\times9}{2} = 9$

SS' = $\frac{4\times7}{2} = 14$

II' = $\frac{4\times7}{2} = 14$

MM' = $\frac{1\times10}{2} = 5$

Total number of ways = $\frac{10\times11}{2} = 55$

Number of ways of letters repeating = 55 – (9 + 14 + 14 + 5) = 13

P(Same) = $\frac{13}{55}$

B1 ${}^{11}C_{2}$ seen as the denominator of fraction (no extra terms) allow unsimplified
M1 1 – sum of 4 appropriate scenarios
A1 Correct final answer

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112. 9709_w18_ms_63 Q: 1

Question	Answer	Marks	Guidance
	Method 1		
	... M … M … M … M … M …	**M1**	k × 5! (120) or k × 6P2 (30), k is an integer ⩾ 1,
	No. ways men placed × No. ways women placed in gaps = $5! \times {}^{6}P_{2}$	**M1**	Correct unsimplified expression
	= 3600	**A1**	Correct answer
	Method 2		
	Number with women together = 6! × 2 (1440) Total number of arrangements = 7! (5040)	**M1**	6! × 2 or 7! – k seen, k is an integer ⩾ 1
	Number with women not together = 7! – 6! × 2	**M1**	Correct unsimplified expression
	= 3600	**A1**	Correct answer
		3	

113. 9709_m17_ms_62 Q: 5

Question	Answer	Marks	Guidance
(i)	${}^{12}C_{1} + {}^{12}C_{3} + {}^{12}C_{5} + {}^{12}C_{7} + {}^{12}C_{9} + {}^{12}C_{11}$	**M1**	Summing at least 4 ${}^{12}C_{x}$ combinations with x = odd numbers
		A1	Correct unsimplified answer (can be implied by final answer)
	= 2048	**A1**	Correct answer
	Total:	**3**	
(ii)	$7! \times {}^{8}P_{4}$	**B1**	7! seen alone or multiplied only (cupcakes ordered)
		M1	multiplying by ${}^{8}P_{4}$ o.e (placing brownies)
	= 8467200	**A1**	correct answer
	Total:	**3**	
(iii)	9! / (6! × 2!)	**B1**	9! oe seen alone or as numerator
		M1	dividing by at least one of 6!,2! (removing repeated shortbread or gingerbread biscuits) ignore 4! if present
	= 252	**A1**	correct answer
	Total:	**3**	

114. 9709_s17_ms_61 Q: 7

Question	Answer	Marks	Guidance
(a)	*EITHER:* e.g. xxxxx =5! for the other children	**(B1**	5! OE seen alone or mult by integer k ⩾ 1, no addition
	Put y in 6 ways, then 5 then 4 for the youngest children	**B1**	Mult by 6P3 OE
	Answer 5! × 6P3 = 14400	**B1)**	Correct answer
	OR: total – 3 tog – 2 tog = 8! – 6!3! – 6! × 2 × 5 × 3 = 14400	**(B1**	8! – 6! × k ⩾ 1seen
		B1	6!3! or 6! × 2 × 5 × 3 seen subtracted
		B1)	Correct answer
	Total:	**3**	
(b)	D W M 2 2 1 = 6C2 × 4C2 × 1 = 90	**B1**	One correct unsimplified option
	3 1 1 = 6C3 × 4 × 1 = 80	**M1**	Summing 2 or more 3-factor options which can contain perms or 3 factors added. The 1 can be implied
	1 3 1 = 6 × 4C3 × 1 = 24	**M1**	Summing the correct 3 unsimplified outcomes only
	Total=194 ways	**A1**	
	Total:	**4**	

Question	Answer	Marks	Guidance
(c)	C D S 2 1 1 = $^{26}C_2 \times 9 \times 5 \times 4!$ = 351 000	**M1**	summing 2 or more options of the form (2 1 1), (1 2 1), (1 1 2), can have perms, can be added
	1 2 1 = $26 \times {}^{9}C_2 \times 5 \times 4!$ = 112 320	**M1**	4 relevant products seen excluding 4! e.g. 26 × 9 × 8 × 5 or 26 × $^{9}P_2$ × 5 for 2nd outcome, condone 26 × 9 × 5 × 37 as being relevant
	1 1 2 = $26 \times 9 \times {}^{5}C_2 \times 4!$ = 56 160	**M1**	mult all terms by 4! or 4!/2!
	Total = 519 480	**A1**	
	Total:	**4**	

115. 9709_s17_ms_62 Q: 6

(i)	*EITHER:* Route 1 *A*********A* in 9! / 2!2!5! = 756 ways	**(*M1**	*Considering AA and BB options* with values
	*B*********B* in 9! / 4!5! = 126 ways	**A1**	Any one option correct
	756 + 126	**DM1**	*Summing their AA and BB outcomes* only
	Total = 882 ways	**A1)**	

Question	Answer	Marks	Guidance
	OR1: Route 2 *A*********A* in $^{9}C_5 \times {}^{4}C_2$ = 756 ways	**(M1**	*Considering AA and BB options* with values
	*B*********B* in $^{9}C_4 \times {}^{5}C_5$ = 126 ways	**A1**	Any one option correct
	756 + 126	**DM1**	*Summing their AA and BB outcomes* only
	Total = 882	**A1)**	
	Total:	**4**	

Question	Answer	Marks	Guidance
(ii)	*EITHER:* (The subtraction method) *A*s together, no restrictions 8! / 2!5! = 168	**(*M1**	*Considering all As together* – 8! seen alone or as numerator – condone × 4! for thinking A's not identical
	*A*s together and *B*s together 7! / 5! = 42	**M1**	*Considering all As together and all Bs together* – 7! seen alone or numerator
		M1	*Removing repeated Bs or Cs* – Dividing by 5! either expression or 2! 1st expression only – OE
	Total 168 – 42	**DM1**	Subt their 42 from their 168 (dependent upon **first M** being awarded)
	= 126	**A1)**	
	OR1: *A*s together, no restrictions $^{8}C_{5}$ x $^{3}C_{1}$ = 168	**(*M1**	$^{8}C_{5}$ seen alone or multiplied
		M1	$^{7}C_{5}$ seen alone or multiplied
	*A*s together and *B*s together $^{7}C_{5}$ x $^{2}C_{1}$ = 42	**M1**	First expression x $^{3}C_{1}$ or second expression x $^{2}C_{1}$
	Total 168 – 42	**DM1**	Subt their 42 from their 168 (dependent upon **first M** being awarded)
	= 126	**A1)**	
	OR2: (The intersperse method)	**(M1**	*Considering all "As together" with Cs* – Mult by 6!
	(*AAAA)CCCCC* then intersperse *B* and another *B*	**M1**	*Removing repeated Cs* – Dividing by 5!– [Mult by 6 implies M2]
		***M1**	*Considering positions for Bs* – Mult by 7P2 oe –
	$\frac{6!}{5!}\times 7\times 6\div 2$	**DM1**	Dividing by 2! Oe – removing repeated *B*s (dependent upon **3rd M** being awarded)
	= 126	**A1)**	
	Total:	**5**	

116. 9709_w17_ms_61 Q: 6

Question	Answer	Marks	Guidance
(a)(i)	$^{40}P_5$	M1	$^{40}P_x$ or $^{y}P_5$ oe seen, can be mult by $k \geqslant 1$
	= 78 960 960	A1	
		2	
(a)(ii)	not front row e.g. WEJ** in 3× 3! = 18 ways	B1	3! seen mult by $k \geqslant 1$
	7 rows in 7 × 18= 126 ways	B1	mult by 7
	front row: e.g. *MA** in 4 × 2 = 8 ways	M1	attempt at front row arrangements and multiplying by the 7 other rows arrangements, need not be correct
	Total 126×8 = 1008	A1	
		4	
(b)	*EITHER:* e.g. *R** in $^{8}C_3$ ways = 56 ways *L** in $^{8}C_3$ = 56 ways	(M1	Considering either R or L only in team
	**** in $^{8}C_4$ = 70 ways	M1*	Considering neither in team
		DM1	summing 3 scenarios
	Total 182 ways	A1)	
	OR1: No restrictions $^{10}C_4$ = 210 ways	(M1	$^{10}C_4$ – , Considering no restrictions with subtraction
	RL = $^{8}C_2$ = 28	M1*	Considering both in team
	210 – 28	DM1	subt
	= 182 ways	A1)	

Question	Answer	Marks	Guidance
(b)	*OR2:* R out in $^{9}C_4$ = 126 ways L out in $^{9}C_4$ = 126 ways	(M1	Considering either R out or L out
	Both out in $^{8}C_4$ = 70	M1*	Considering both out
		DM1	Summing 2 scenarios and subtracting 1 scenario
	126 + 126 – 70 = 182 ways.	A1)	
		4	

117. 9709_w17_ms_62 Q: 6

Question	Answer	Marks	Guidance
(a)(i)	*EITHER:* 3**, 4**, 6**, 8**	(M1	$^{5}P_2$ or $^{5}C_2 \times 2!$ or 5 × 4 OE (considering final 2 digits)
	options 4 × 5 × 4 = 80	M1	Mult by 4 or summing 4 options (considering first digit)
		A1)	Correct final answer
	OR: Total number of values: 6 × 5 × 4 = 120	(M1	Calculating total number of values (with subtraction seen)
	Number of values less than 300: 2 × 5 × 4 = 40	M1	Calculating number of unwanted values
	Number of evens = 120 – 40 = 80	A1)	Correct final answer
		3	

Question	Answer	Marks	Guidance
(a)(ii)	3**, 4**, 6**, 8** *EITHER:* options 4 × 6 × 4 (last)	(M1	6 linked to considering middle digit e.g. multiplied or in list
		M1	Multiply an integer by 4 × 4 (condone × 16) (No additional figures present for both M's to be awarded)
	= 96	A1)	
	OR: Total number of values 4 × 6 × 6 = 144	(M1	Calculating total number of values (with subtraction seen)
	Number of odd values 4 × 6 × 2 = 48	M1	Calculating number of unwanted values
	Number of evens = 144 – 48 = 96	A1)	
		3	
(b)(i)	252	B1	
		1	

Question	Answer	Marks	Guidance
(b)(ii)	B (6)G(4)		
	5 0 in ${}^6C_5$ (×${}^4C_0$) = 6 × 1 = 6 4 1 in ${}^6C_4$ × ${}^4C_1$ = 15 × 4 = 60 3 2 in ${}^6C_3$ × ${}^4C_2$ = 20 × 6 = 120	M1	Multiplying 2 combinations ${}^6C_q \times {}^4C_r$, $q + r = 5$, or ${}^6C_5$ seen alone
		M1	Summing 2 or 3 appropriate outcomes, involving perm/comb, no extra outcomes.
	Total = 186 ways	A1	
		3	

118. 9709_m16_ms_62 Q: 6

(i)	${}^{15}P_5$ = 360360	**M1** **A1** 2	oe, can be implied Not ${}^{15}C_5$ Correct answer
(ii)	5 × 10 × 4 × 9 × 3 = 5400	**M1** **A1** 2	Mult 5 numbers Correct answer
(iii)	M(5) F(10) 3 2 = ${}^5C_3 \times {}^{10}C_2$ = 450 ways 4 1 = ${}^5C_4 \times {}^{10}C_1$ = 50 5 0 = ${}^5C_5 \times {}^{10}C_0$ = 1 Total = 501 ways	**M1** **M1** **A1** 3	Mult 2 combs, ${}^5C_x \times {}^{10}C_y$ Summing 2 or 3 two-factor options, $x + y = 5$ Correct answer
(iv)	(Couple) M(4) F(9) ManWife + 3 0 = ${}^4C_3 \times {}^9C_0 = 4$ ManWife + 2 1 = ${}^4C_2 \times {}^9C_1 = 54$ Total = 58	**M1** **M1** **A1** 3	Mult 2 combs ${}^4C_x$ and ${}^9C_y$ Summing both options $x + y = 3$, gender correct Correct answer

119. 9709_s16_ms_61 Q: 6

(a) (i)	$9 \times 9 \times 8$	**M1 M1**	Logical listing attempt
	$= 648$	**A1** [3]	
	OR $900 - 28 \times 9 = 648$		
(ii)	(7….in $1 \times 8 \times 4 = 32$ ways	**M1**	Listing #s starting with 7 or 9 and ending odd
	8 …in $1 \times 8 \times 5 = 40$	**M1**	
	9… in $1 \times 8 \times 4 = 32$	**M1**	
	Total 104 ways	**A1** [4]	
(b)	R(6) T(5) D(4)		
	2 2 3 = ${}^6C_2 \times {}^5C_2 \times {}^4C_3 = 600$	**M1**	Mult 3 combs, ${}^6C_x \times {}^5C_y \times {}^4C_z$
	2 3 2 = ${}^6C_2 \times {}^5C_3 \times {}^4C_2 = 900$	**M1**	Summing 2 or 3 three-factor outcomes can be perms, + instead of ×
	3 2 2 = ${}^6C_3 \times {}^5C_2 \times {}^4C_2 = 1200$	**A1**	2 options correct unsimplified
	Total = 2700	**A1** [4]	

120. 9709_s16_ms_62 Q: 7

(a) (i)	$\frac{10!}{2!3!} = 302400$	**B1** [1]	Exact value only, isw rounding

(ii)	e.g. *W******W*, **W******W, W******W**	**M1**	8! Seen mult or alone. Cannot be embedded (arrangements of other 8 letters).
	$\frac{8!}{3!} \times 3$(for the Ws)	**M1**	Dividing by 3! (removing repeated L's)
		M1	Mult by 3 (different W positions) may be sum of 3 terms
	$= 20160$	**A1** [4]	
(b)	S(5) A(7) C(4) 1 3 2 : $5 \times {}^7C_3 \times {}^4C_2 = 1050$ 1 4 1 : $5 \times {}^7C_4 \times 4 = 700$ 2 3 1 : ${}^5C_2 \times {}^7C_3 \times 4 = 1400$ 3 2 1 : ${}^5C_3 \times {}^7C_2 \times 4 = 840$ (Outcomes : Options)	**M1**	Mult 3 combinations, ${}^5C_x$, ${}^7C_y$, ${}^4C_z$ (not 5 x 7 x 4)
		A1	2 correct options unsimplified
		M1	Summing only 3 or 4 correct outcomes involving combs or perms
	Total = 3990	**A1** [4]	

121. 9709_s16_ms_63 Q: 6

Qu	Answer	Marks	Guidance
(i)	7560 ways	B1 [1]	
(ii)	RxxxxxxxG in $\frac{7!}{4!}$	B1	7! alone seen in num or 4! alone in denom Must be in a fraction. $\frac{7!\times 2}{4!\times 2}$ gets full marks
	= 210 ways	B1 [2]	
(iii)	eg EEEExxxxx in $\frac{6!}{2!}$	B1	6! or 5! × 6 seen in numerator or on own Can be 6! × k but not 6! ± k
	= 360 ways	B1 [2]	
(iv)	1 R eg RVG or RVN or RGN = 3	B1 [1]	
(v)	no Rs eg VGN or 3C3 ways = 1 2 Rs eg RRV or 3C1 ways = 3	M1	Summing at least 2 options for R
	Total = 7	A1 A1 [3]	Correct outcome for no Rs or 2 Rs – evaluated

122. 9709_w16_ms_61 Q: 5

(a)	e.g. P*N*P*P*L	M1		Mult by 5! in num
	$= \frac{5!}{3!}\times\frac{{}^6P_4}{2!}$	M1 M1		Dividing by 3! or 2! Mult by ${}^6P_4$ oe
	= 3600	A1	[4]	
(b) (i)	${}^7C_5 \times {}^5C_4 \times {}^2C_1 \times {}^2C_1$	M1		Mult 4 combs of which three are correct
	= 420	A1	[2]	
(ii)	both in team	M1		Evaluating both in team and subtracting from (i)
	${}^6C_4 \times {}^4C_3 \times 2 \times 2 = 240$	M1		240 seen can be unsimplified ft their 420, their 240
	420 – 240 = 180 ways	A1		
	OR			
	Bat in bowl out + bowl in bat out + both out	M1		summing 2 or 3 options not both in team
	$= {}^6C_4\times{}^4C_3\times2\times2+{}^6C_5\times{}^4C_3\times2\times2+{}^6C_5\times{}^4C_4\times2\times2$	A1		2 or 3 options correct unsimplified
	= 60 + 96 + 24 = 180 ways	A1		Correct ans from correct working
	OR			
	Bat in bowl out + bat out	M1		As above, or bowl in bat out + bowl out
	$= 60 + {}^6C_5\times{}^5C_4\times2\times2 = 60 + 120 = 180$ ways	A1 A1	[3]	

123. 9709_w16_ms_62 Q: 6

(i)	e.g. (OAEE)(CPNHGN) or cv $\frac{4!}{2!}\times\frac{6!}{2!}\times 2 = 8640$	**M1** **M1** **A1**	[3]	4!/2! or 6!/2! seen anywhere All multiplied by 2 oe
(ii)	First Method Total ways = 10!/2!2! = 907200 EE together in 9!/2! ways = 181440 EE not together = 907200 – 181440 = 725760 **OR** Second Method C P N H G N O A in 8!/2! ways Insert E in 9 ways Insert 2nd E in 8 ways, ÷2 Total = 8!/2!×9×8÷2 = 725760	**B1** **M1** **M1** **A1** **B1** **M1** **M1** **A1**	[4]	Total ways together correct EE together attempt alone Considering total – EE together 8!/2! Seen Interspersing an E, x n where n=7,8,9. Condone additional factors. Mult by 9×8(÷2), $^9C_2$ or $^9P_2$ only oe
(iii)	First Method EN** in $^6C_2$ ways = 15 different ways EENN in 1 way Total 16 ways **OR** Second Method Listing with at least 8 different correct options Listing all correct options Total = 15 different ways EENN in 1 way Total 16 ways	**M1** **M1** **A1** **B1** **A1** **M1** **M1** **A1** **B1** **A1**	[5]	^6C_x or $^yC_2$ seen alone or mult by $k > 1$, x<6, y>2 (1x1x) $^6C_2$ seen strictly alone or added to their EENN only Value stated or implied by final answer correct value stated Award 16 SRB2 if no method is present

124. 9709_w16_ms_63 Q: 1

	total ways $^{10}C_5 = 252$ MW together e.g. (MW)*** in $^8C_3$ ways = 56 MW not together = 252 – 56 = 196 ways **OR 1** $2\ ^8C_4 + {}^8C_5$ $2\ ^8C_4 = 2x70 = 140$; $^8C_5 = 56$ $2\ ^8C_4 + {}^8C_5 = 196$ **OR 2** $2\ ^9C_5 - {}^8C_5$ $2\ ^9C_5 = 2 \times 126 = 252$; $^8C_5 = 56$ $2\ ^9C_5 - {}^8C_5 = 196$	**M1** **B1** **A1** **M1** **B1** **A1** **M1** **B1** **A1**	[3]	$^{10}C_5 - \ldots$ or $252 - \ldots$ 252 and 56 seen, may be unsimplified $2\ ^nC_4 + {}^nC_5$ 140 and 56 seen may be unsimplified $2\ ^9C_5 - ..$ 252 and 56 seen, may be unsimplified

125. 9709_w16_ms_63 Q: 3

(i)	e.g. **5 in $^3P_2$ ways = 6	**M1**		Recognising ends in 5 or 7, can be implied
	**7 in $^3P_2$ = 6	**M1**		Summing ends in 5 + ends in 7 oe
	Total 12 AG	**A1**	[3]	Correct answer following legit working
	OR listing 457, 547, 467, 647, 567, 657, 475, 745 465, 645, 675, 765	**M1**		Listing at least 5 different numbers ending in 5
		M1		Listing at least 5 different numbers ending in 7
	Total 12 AG	**A1**		
(ii)	1 digit in 2 ways 2 digits in *5 or *7 = $^3P_1 \times 2 = 6$	**M1**		Consider at least 3 options with different number of digits. If no working, must be 3 or 4 from 2, 6, 12, 12
	4 digits in ***5 or ***7 = $^3P_3 \times 2 = 12$	**A1**		One option correct from 1, 2 or 4 digits
	Total ways = 32	**A1**	[3]	

126. 9709_s15_ms_61 Q: 7

(a) (i)	$\frac{9!}{2!2!3!}$	B1		Dividing by 2!2!3!
	= 15120 ways	B1	[2]	Correct answer

(ii)	********3 in $\frac{8!}{2!2!3!}$ = 1680 ways	B1		Correct ways end in 3
	********7 in $\frac{8!}{2!3!}$ = 3360 ways	B1		Correct ways end in 7
	Total even = 15120 – 1680 – 3360	M1		Finding odd and subt from 15120 or their **(i)**
	= 10080 ways	A1	[4]	Correct answer
	OR ********2 in 8!/2!3! = 3360 ways	B1		One correct way end in even
	********6 in 8!/2!2!3! = 1680 ways	B1		correct way end in another even
	********8 in 8!/2!2!2! = 5040ways	M1		Summing 2 or 3 ways
	Total = 10080 ways	A1		Correct answer
	OR "15120" ×6/9 = 10080	M2		Mult their **(i)** by 2/3 oe
		A2		Correct answer
(b)	T(3) S(6) G(14) 1 1 3 in $3\times6\times{}^{14}C_3 = 6552$	M1		Mult 3 (combinations) together assume $6 = {}^6C_1$etc
	1 3 1 in $3\times{}^6C_3\times14 = 840$			
	3 1 1 in $1\times6\times14 = 84$	M1		Listing at least 4 different options
	2 2 1 in $^3C_2\times{}^6C_2\times14 = 630$	M1		Summing at least 4 different options
	2 1 2 in $^3C_2\times6\times{}^{14}C_2 = 1638$			
	1 2 2 in $3\times{}^6C_2\times{}^{14}C_2 = 4095$	B1		At least 3 correct numerical options
	Total ways = 13839 (13800)	A1	[5]	Correct answer

127. 9709_s15_ms_62 Q: 6

(a)	**(i)**	N*****B Number of ways = $\frac{5!}{3!}$ = 20	B1 B1 B1 **3**	5! Seen in num oe or alone mult by $k \geqslant 1$ 3! Seen in denom can be mult by $k \geqslant 1$ Correct final answer
	(ii)	B(AAA)NNS Number of ways = $\frac{5!}{2!}$ or ${}^5P_3$ = 60	M1 M1 A1 **3**	5! seen as a num can be mult by $k \geqslant 1$ Dividing by 2! Correct final answer
(b)		${}^{14}C_9$ total options = 2002 T and M both in ${}^{12}C_7 = 792$ Ans 2002 – 792 = 1210 OR Neither in ${}^{12}C_9 = 220$ One in ${}^{12}C_8 = 495$ Other in ${}^{12}C_8 = 495$	M1 B1 A1 **3** M1 B1	${}^{14}C_9$ or ${}^{14}P_9$ in subtraction attempt ${}^{12}C_7$ (792) seen Correct final answer Summing 2 or 3 options at least 1 correct condone ${}^{12}P_9 + {}^{12}P_8 + {}^{12}P_8$ here only Second correct option seen accept another 495 or if M1 not awarded, any correct option

128. 9709_s15_ms_63 Q: 7

(i)		W S D 1 1 3 = $6\times4\times{}^3C_3 = 24$ 1 3 1 = $6\times{}^4C_3\times3 = 72$ 3 1 1 = ${}^6C_3\times4\times3 = 240$ 1 2 2 = $6\times{}^4C_2\times{}^3C_2 = 108$ 2 1 2 = ${}^6C_2\times4\times{}^3C_2 = 180$ 2 2 1 = ${}^6C_2\times{}^4C_2\times3 = 270$ Total = 894	M1 M1 M1 B1 A1 **[5]**	Listing at least 4 different options Mult 3 (combs) together assume 6 = ${}^6C_1$, Σr=5 Summing at least 4 different evaluated/unsimplified options >1 At least 3 correct unsimplified options Correct answer
(ii)		${}^3P_2 \times {}^{10}P_8$ = 10886400	B1 B1 B1 **[3]**	${}^3P_2$ oe seen multiplied either here or in (iii) $k^{10}P_x$ seen or k^yP_8 with no addition, $k\geqslant1$, $y>8$, $x<10$ Correct answer, nfww
(iii)		DSWSWSWSWD or DWSWSWSWSD D in ${}^3P_2$ ways = 6 S in ${}^4P_4$ ways = 24 W in ${}^6P_4 = 360$ Swap SW in 2 ways Total = 103680 ways	 B1 B1 B1 **[3]**	If ${}^3P_2$ has not gained credit in (ii) may be awarded ${}^4P_4$ or ${}^6P_4$ oe seen multiplied or common in all terms (no division) Mult by 2 (condone 2!) Correct answer, 3sf or better, nfww

129. 9709_w15_ms_61 Q: 5

(i)	**5 (i)** eg **(EEEE)*** Number of ways = $\frac{6!}{2!2!} = 180$	**M1** **M1** **A1** [3]	Mult by 6! oe Dividing by 2!2! oe Correct answer
(ii)	S*******T or T*******S Number of ways = $\frac{7!}{4!2!} \times 2$ = 210	**M1** **M1** **A1** [3]	Mult by 7! Or dividing by one of 2! or 4! Mult by 2 Correct answer
(iii)	exactly one E in ${}^6C_3$ ways = 20	**M1** **M1** **A1** [3]	${}^6C_x$ as a single answer ${}^xC_3$ as a single answer correct answer

130. 9709_w15_ms_62 Q: 4

(i)	Two in same taxi: ${}^6C_2 \times {}^4C_4 \times 2$ or ${}^6C_2 + {}^6C_4$ = 30	**M1** **M1** **A1** 3	${}^6C_4$ or ${}^6C_2$ oe seen anywhere 'something' ×2 only or adding 2 equal terms Correct final answer
(ii)	MJS in taxi $({}^5C_1 \times 2 \times 2) \times {}^4P_4$ = 480	**M1** **M1** **M1** **A1** 4	${}^5P_1$, ${}^5C_1$ or 5 seen anywhere Mult by 2 or 4 oe Mult by ${}^4P_4$ oe eg 4! or $4 \times {}^3P_3$ or can be part of 5! Correct final answer

131. 9709_w15_ms_63 Q: 5

(a)	e.g. **(AAOOOI)***** $\frac{8!}{2!2!} \times \frac{6!}{2!3!} = 604800$	**B1** **M1** **A1** 3	8! (8 × 7!) or 6! seen anywhere, either alone or in numerator) Dividing by at least 3 of 2!2!2!3! (may be fractions added) Correct answer
(b)	C(7) E(6) A(4) 1 1 2 $= 7 \times 6 \times {}^4C_2 = 252$ 1 2 1 $= 7 \times {}^6C_2 \times 4 = 420$ 1 3 0 $= 7 \times {}^6C_3 \times 1 = 140$ 2 1 1 $= {}^7C_2 \times 6 \times 4 = 504$ 2 2 0 $= {}^7C_2 \times {}^6C_2 \times 1 = 315$ 3 1 0 $= {}^7C_3 \times 6 \times 1 = 210$ Total = 1841	**M1** **A1** **M1*** **DM1** **A1** 5	Mult 3 appropriate combinations together assume $6={}^6C_1$, $1={}^4C_0$ etc., $\sum r=4$, C&E both present At least 3 correct unsimplified products Listing at least 4 different correct options Summing at least 4 outcomes, involving 3 combs or perms, $\sum r=4$ Correct answer SC if CE removed, M1 available for listing at least 4 different correct options for remaining 2. DM1 for ${}^7C_1 \times {}^6C_1 \times$(sum of at least 4 outcomes)

132. 9709_s22_ms_51 Q: 6

Question	Answer	Marks	Guidance
(a)	$0.6 + 0.4 \times 0.3 = 0.72$ or $1 - 0.4 \times 0.7 = 0.72$	B1	Clear identified calculation AG
		1	
(b)	$0.72 \times (0.4 + 0.6 \times 0.2)$	M1	$0.72 \times u$, $0 < u < 1$
		M1	$v \times (0.4 + 0.6 \times 0.2)$, or $v \times (1 - 0.6 \times 0.8)$ $0 < v \leqslant 1$ no additional terms SC B1 for $0.72 \times (0.4 + 0.12)$ or $0.72 \times (1 - 0.48)$
	0.3744	A1	WWW. Condone 0.374. SC B1 for 0.3744 only
		3	
	Alternative method for question 6(b)		
	[p(P1P2) + p(F1P1P2) + p(P1F2P2) + p(F1P1F2P2)] = $0.6 \times 0.4 + 0.4 \times 0.3 \times 0.4 + 0.6 \times 0.6 \times 0.2 + 0.4 \times 0.3 \times 0.6 \times 0.2$	M1	Any two terms unsimplified and correct
		M1	Summing 4 appropriate scenarios by listing or on a tree diagram SC B1 for 0.24 + 0.048 + 0.072 + 0.0144
	0.3744	A1	WWW. Condone 0.374. SC B1 for 0.3744 only
		3	

Question	Answer	Marks	Guidance
(c)	$P(\text{fails first or second level} \mid \text{finishes game}) = \frac{P(\text{fails first or second level} \cap \text{finishes game})}{\textit{their}\ \textbf{(b)}}$	M1	Either $0.6 \times 0.6 \times 0.2$ or $0.4 \times 0.3 \times 0.4$ seen Condone 0.072 or 0.048 if seen in **(b)**
	Numerator = P(S SF) + P(FS S) = $0.6 \times 0.6 \times 0.2 + 0.4 \times 0.3 \times 0.4 = 0.072 + 0.048 = 0.12$	A1	Both correct accept unsimplified expression. No additional terms
	Required probability = $\frac{0.12}{\textit{their}\ \textbf{(b)}}$	M1	$\frac{\textit{Their}\ \text{sum of two 3-term probabilities as numerator}}{\textit{their}\ \textbf{(b)}\ \text{or correct}}$
	0.321 or $\frac{25}{78}$	A1	$0.3205 < p \leqslant 0.321$
		4	

133. 9709_s22_ms_52 Q: 7

Question	Answer	Marks	Guidance
(a)	YYY: $\frac{5}{12} \times \frac{4}{11} \times \frac{3}{10} = \frac{60}{1320}, \frac{1}{22}$ OOO: $\frac{4}{12} \times \frac{3}{11} \times \frac{2}{10} = \frac{24}{1320}, \frac{1}{55}$ RRR: $\frac{3}{12} \times \frac{2}{11} \times \frac{1}{10} = \frac{6}{1320}, \frac{1}{220}$	M1	**Either** $12 \times 11 \times 10$ in denominator **or** $a \times (a-1) \times (a-2)$, $a = 5, 4, 3$ in numerator seen in at least one expression.
		A1	One expression $\frac{a}{12} \times \frac{a-1}{11} \times \frac{a-2}{10}$, $a = 5, 4, 3$ (consistent in expression). Correct order of values in the numerator is essential.
		M1	$\frac{5}{12} \times \frac{4}{d} \times \frac{3}{e} + \frac{4}{12} \times \frac{3}{d} \times \frac{2}{e} + \frac{3}{12} \times \frac{2}{d} \times \frac{1}{e}$, either $d = 11$, $e = 10$ or $d = 12$, $e = 12$. Condone $\frac{1}{22} + \frac{1}{55} + \frac{1}{220}$ OE
	[Total =] $\frac{90}{1320}, \frac{3}{44}, 0.0682$	A1	0.06818. Dependent only upon the second M mark.

Question	Answer	Marks	Guidance
(a)	**Alternative method for question 7(a)**		
	YYY: $\frac{{}^5C_3}{{}^{12}C_3} = \frac{10}{220}, \frac{1}{22}$ OOO: $\frac{{}^4C_3}{{}^{12}C_3} = \frac{4}{220}, \frac{1}{55}$ RRR: $\frac{{}^3C_3}{{}^{12}C_3} = \frac{1}{220}$	**M1**	Either ${}^{12}C_3$ in denominator or ${}^aC_3$ in numerator seen in at least one expression.
		A1	One expression $\frac{{}^aC_3}{{}^{12}C_3}$ $a = 5, 4, 3$
		M1	$\frac{{}^5C_3}{{}^{12}C_3} + \frac{{}^4C_3}{{}^{12}C_3} + \frac{{}^3C_3}{{}^{12}C_3}$ Condone $\frac{1}{22} + \frac{1}{55} + \frac{1}{220}$ OE
	[Total =] $\frac{90}{1320}, \frac{3}{44}, 0.0682$	**A1**	0.06818. Dependent only upon the second M mark.
		4	
(b)	[P(YYY | all same colour) =] $\frac{60}{1320} \div \frac{90}{1320}$	**M1**	$\frac{\textit{their}\,\mathrm{P(YYY)} \text{ or } \frac{60}{1320} \text{ or } \frac{1}{22}}{\textit{their}\,7(\mathrm{a}) \text{ or } \frac{90}{1320} \text{ or } \frac{3}{44}}$
	$\frac{2}{3}$, 0.667	**A1**	OE
		2	

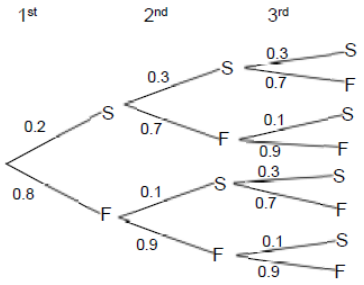
Question	Answer	Marks	Guidance
(c)	In each method, the M mark requires the scenarios to be identifiable. This may be implied by a list of scenarios and then the calculations which will be assumed to be in the same order. A correct value/expression will be condoned as identifying the connected scenario.		
	Method 1		
	[1 – no orange =] $1 - \frac{8}{12} \times \frac{7}{11} \times \frac{6}{10}$ or $1 - \frac{{}^8C_3}{{}^{12}C_3} = 1 - \frac{14}{55}$	**B1**	$\frac{8}{12} \times \frac{7}{11} \times \frac{6}{10}$ or $\frac{{}^8C_3}{{}^{12}C_3}$ seen, condone $\frac{336}{1320}$ or $\frac{56}{220}$ only, not OE.
		M1	$1 - \frac{f}{12} \times \frac{g}{d} \times \frac{h}{e}$ Either $d = 11$, $e = 10$ or $d = 12$, $e = 12$ or $1 - \frac{{}^8C_3}{{}^{12}C_3}$. Condone $1 - \frac{14}{55}$ OE (not $\frac{41}{55}$).
	$\frac{41}{55}$	**A1**	$0.745 \leqslant p \leqslant 0.74545$ If M0 scored **SC B1** $0.745 \leqslant p \leqslant 0.74545$.

Question	Answer	Marks	Guidance
(c)	**Method 2**		
	$P(1\,O)=\left(\frac{4}{12}\times\frac{3}{11}\times\frac{2}{10}+\frac{4}{12}\times\frac{5}{11}\times\frac{4}{10}+2\times\frac{4}{12}\times\frac{5}{11}\times\frac{3}{10}\right)\times 3=\frac{672}{1320}$ $P(2O)=\frac{4}{12}\times\frac{3}{11}\times\frac{8}{10}\times 3=\frac{288}{1320}$ $P(3O)=\frac{24}{1320}$	**B1**	P(1 O)or P(2 O) correct, accept unsimplified.
		M1	3 correct scenarios added, with at least one 3-term product of form $\frac{f}{12}\times\frac{g}{d}\times\frac{h}{e}$ seen, either $d=11$, $e=10$ or $d=12$, $e=12$.
	[Total =] $\frac{984}{1320}=\frac{41}{55}$, 0.745	**A1**	$0.745\leqslant p\leqslant 0.74545$ If M0 scored **SC B1** $0.745\leqslant p\leqslant 0.74545$.
	Method 3		
	O Y R = ${}^4C_1\times{}^5C_1\times{}^3C_1$ = 60 O R R = ${}^4C_1\times{}^3C_2$ = 12 O Y Y = ${}^4C_1\times{}^5C_2$ = 40 O O Y = ${}^4C_2\times{}^5C_1$ = 30 O O R = ${}^4C_2\times{}^3C_1$ = 18 O O O = ${}^4C_3$ = 4 Total = 164 Prob = $\frac{164}{{}^{12}C_3}$	**B1**	Number of ways either 1 or 2 orange sweets obtained correctly (112 or 48). Accept unsimplified Note ${}^4C_1\times{}^8C_2=112$ or ${}^4C_2\times{}^8C_1=48$ are correct alternatives.
		M1	3 correct scenarios (1, 2 or 3 orange sweets) added on numerator, denominator ${}^{12}C_3$
	$\frac{984}{1320}=\frac{41}{55}$, 0.745	**A1**	$0.745\leqslant p\leqslant 0.74545$ If M0 scored **SC B1** $0.745\leqslant p\leqslant 0.74545$.

Question	Answer	Marks	Guidance
(c)	**Method 4**		
	$P(R\,R\,O)=\frac{3}{12}\times\frac{2}{11}\times\frac{4}{10}=\frac{1}{55}$ $P(R\,R\,\hat{}\,)=\frac{3}{12}\times\frac{4}{11}=\frac{1}{11}$ $P(R\,Y\,O)=\frac{3}{12}\times\frac{5}{11}\times\frac{4}{10}=\frac{1}{22}$ $P(O\,\hat{}\,\hat{}\,)=\frac{4}{12}=\frac{1}{3}$ $P(Y\,R\,O)=\frac{5}{12}\times\frac{3}{11}\times\frac{4}{10}=\frac{1}{22}$ $P(Y\,R\,\hat{}\,)=\frac{5}{12}\times\frac{4}{11}=\frac{5}{33}$ $P(Y\,Y\,O)=\frac{5}{12}\times\frac{4}{11}\times\frac{4}{10}=\frac{2}{33}$	**B1**	$P(R\,\hat{}\,\hat{}\,)=\frac{17}{110}$ or $P(Y\,\hat{}\,\hat{}\,)=\frac{17}{66}$. Accept unsimplified.
		M1	3 correct scenarios added, with at least one 3-term product of form $\frac{f}{12}\times\frac{g}{d}\times\frac{h}{e}$ seen, either $d=11$, $e=10$ or $d=12$, $e=12$.
	$\frac{984}{1320}=\frac{41}{55}$, 0.745	**A1**	$0.745\leqslant p\leqslant 0.74545$ If M0 scored **SC B1** $0.745\leqslant p\leqslant 0.74545$.

Question	Answer	Marks	Guidance
(c)	**Method 5**		
	$P(O\,\hat{}\,\hat{}\,)=\frac{4}{12}=\frac{1}{3}$ $P(\hat{}\,O\,\hat{}\,)=\frac{8}{12}\times\frac{4}{11}=\frac{8}{33}$ $P(\hat{}\,\hat{}\,O)=\frac{8}{12}\times\frac{7}{11}\times\frac{4}{10}=\frac{28}{165}$	**B1**	$P(\hat{}\,O\,\hat{}\,)=\frac{8}{33}$ or $P(\hat{}\,\hat{}\,O)=\frac{28}{165}$. Accept unsimplified.
		M1	3 correct scenarios added, with at least one 3-term product of form $\frac{f}{12}\times\frac{g}{d}\times\frac{h}{e}$ seen, either $d=11$, $e=10$ or $d=12$, $e=12$ with correct numerator.
	$\frac{984}{1320}=\frac{41}{55}$, 0.745	**A1**	$0.745\leqslant p\leqslant 0.74545$ If M0 scored **SC B1** $0.745\leqslant p\leqslant 0.74545$.
		3	

134. 9709_s22_ms_53 Q: 6

Question	Answer	Marks	Guidance
(a)		B1	First and second jumps correct with probabilities and outcomes identified.
		B1	Third jump correct with probabilities and outcomes identified.
		2	
(b)	SFF $0.2\times0.7\times0.9 = 0.126$ FSF $0.8\times0.1\times0.7 = 0.056$ FFS $0.8\times0.9\times0.1 = 0.072$	M1	Two or three correct 3 factor probabilities added, correct or FT from part **6(a)**. Accept unsimplified.
	[Total = probability of 1 success =] $0.254\left(\frac{127}{500}\right)$	A1	Accept unsimplified.
	[Probability of at least 1 success = $1-0.8\times0.9\times0.9$ =]$0.352\left(\frac{44}{125}\right)$	B1 FT	Accept unsimplified.
	P(exactly 1 success | at least 1 success)= $\frac{\textit{their}\,0.254}{\textit{their}\,0.352}$	M1	Accept unsimplified.
	$0.722, \frac{127}{176}$	A1	$0.7215 < p \leqslant 0.722$
		5	

Question	Answer	Marks	Guidance
(c)	$0.8\times0.9\times0.9\times0.1\times0.3\times0.3 = 0.005832$ [FFFSSS] $0.2\times0.3\times0.3\times0.7\times0.9\times0.9 = 0.010206$ [SSSFFF]	M1	$a \times b \times c \times d \times e \times f$ FT from *their* tree diagram. Either a, b and c all = 0.8 or 0.9 (at least one of each) and d, e and f all = 0.1 or 0.3 (at least one of each). Or a, b, c = 0.2 or 0.3 (at least one of each) and d, e, f = 0.7 or 0.9 (at least one of each).
		A1	Either correct. Accept unsimplified.
	[Total =] 0.0160[38]	A1	
		3	

135. 9709_m21_ms_52 Q: 2

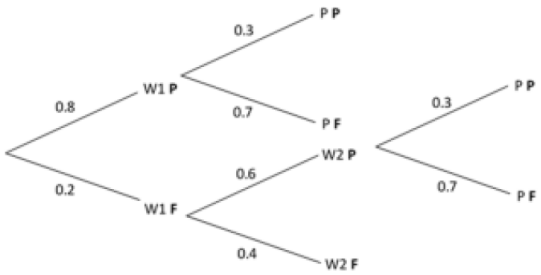
Question	Answer	Marks	Guidance
(a)	$0.2[\times1]+0.45\times0.4+0.35\times0.3$	M1	0·2 [× 1] + 0·45 × b + 0·35 × c, b = 0·4, 0·6 c = 0·3, 0·7
	0·485 or $\frac{97}{200}$	A1	
		2	
(b)	$P(Y\mid\overline{H}) = \frac{P(Y\cap\overline{H})}{P(\overline{H})} = \frac{0.35\times0.7}{1-\textit{their}\textbf{(a)}} = \frac{0.245}{0.515}$	B1	0·35 × 0·7 or 0·245 seen as numerator or denominator of fraction.
		M1	0·515 or 1 – *their* **(a)** or [0·3 × 0 +] 0·45 × d + 0·35 × e, where d = *their b'*, e = *their c'* seen as denominator of fraction.
	0·476 or $\frac{49}{103}$	A1	$0.4757 \leqslant p \leqslant 0.476$
		3	

136. 9709_s21_ms_51 Q: 3

Question	Answer	Marks	Guidance
(a)	$\left[\frac{8!}{3!}\right] = 6720$	B1	NFWW, must be evaluated
		1	
(b)	_ _ _ L E D _ _ : With LED together: $\frac{6!}{2!}$	M1	$\frac{6!}{k}$ or $\frac{5!x6}{k}$ $k \geqslant 1$ and no other terms
		M1	$\frac{m}{2!}$, m an integer, $m \geqslant 5$
	360	A1	CAO
		3	
(c)	Method using _ _ _ A _ D _ _ : Arrange the 6 letters RELESE = $\frac{6!}{3!}$ [= 120]	*M1	$\frac{6!}{3!} \times k$ seen, k an integer > 0
	Multiply by number of ways of placing AD in non-adjacent places = *their* 120 × $^7P_2$ [= 5040]	*M1	$m \times n(n-1)$ or $m \times {}^nC_2$ or $m \times {}^nP_2$, $n = 6$, 7 or 8, m an integer > 0
	[Probability =] $\frac{\textit{their}\ 5040}{\textit{their}\ 6720}$	DM1	Denominator = *their* **(a)** or correct, dependent on at least one M mark already gained.
	$\frac{5040}{6720}$ or $\frac{3}{4}$ or 0.75	A1	
	Alternative method for Question 3(c)		
	Method using 'Total arrangements – Arrangements with A and D together': *Their* 6720 – $\frac{7!\times 2}{3!}$ [= 5040]	*M1	*Their* 6720 – k, k a positive integer
		*M1	$(m-)\frac{7!\times k}{3!}, k = 1, 2$

Question	Answer	Marks	Guidance
	[Probability =] $\frac{\textit{their}\ 5040}{\textit{their}\ 6720}$	DM1	With denominator = *their* **(a)** or correct, dependent on at least one M mark already gained.
	$\frac{5040}{6720}$ or $\frac{3}{4}$ or 0.75	A1	
	Alternative method for Question 3(c)		
	Method using '1 – Probability of arrangements with A and D together': $\frac{7!\times 2}{3!}$ [= 1680]	*M1	$\frac{7!\times k}{3!}, k = 1, 2$
	[Probability =] $\frac{\textit{their}\ 1680}{\textit{their}\ 6720}$	*M1	With denominator = *their* **(a)** or correct
	$1 - \frac{\textit{their}\ 1680}{\textit{their}\ 6720}$	DM1	$1 - m$, $0 < m < 1$, dependent on at least one M mark already gained
	$\frac{5040}{6720}$ or $\frac{3}{4}$ or 0.75	A1	
		4	

137. 9709_s21_ms_51 Q: 4

Question	Answer	Marks	Guidance
(a)		B1	Fully correct labelled tree diagram for each pair of branches clearly identifying written and practical, pass and fail for each intersection (no additional branches)
		B1	'One written test' branch all probabilities (or %) correct
		B1	'Two written tests' branch all probabilities (or %) correct, condone additional branches after W2F with probabilities 1 for PF and 0 for PP
		3	
(b)	[P(W1P) × P(PP) + P(W1F) × P(W2P) × P(PP)] $0.8\times0.3+0.2\times0.6\times0.3$	M1	Consistent with *their* tree diagram or correct
	0.276 or $\frac{69}{250}$	A1	
		2	
(c)	$P(W1\|P)=\frac{P(W1\cap\text{Practical})}{P(\text{getting place})}=\frac{0.8\times0.3}{their\,(b)}\left[=\frac{0.24}{0.276}\right]$	M1	Correct expression or FT *their* **(b)**
	$\frac{20}{23}$ or 0.87[0]	A1	
		2	

138. 9709_s21_ms_52 Q: 3

Question	Answer	Marks	Guidance
(a)	P(not late) = 0.4 × 0.45 + 0.35 × 0.3 + 0.25 × (1 – x) or P(late) = 0.4 × 0.55 + 0.35 × 0.7 + 0.25x	M1	$0.4\times p+0.35\times q+0.25\times r$, p = 0.45, 0.55, q = 0.3, 0.7 and r = (1 – x), x
	0.18 + 0.105 + 0.25 (1 – x) = 0.48 or 0.22 + 0.245 + 0.25x = 0.52	A1	Linear equation formed using sum of 3 probabilities and 0.48 or 0.52 as appropriate. Accept unsimplified.
	x = 0.22	A1	Final answer
		3	
(b)	$\left[P(train\|late)=\frac{P(train\cap late)}{P(late)}\right]$ $=\frac{0.35\times0.7}{1-0.48}$ or $\frac{0.35\times0.7}{0.4\times0.55+0.35\times0.7+0.25\times their\,0.22}$	B1	0.35×0.7 or 0.245 seen as numerator of fraction
		M1	P(late) seen as a denominator with *their* probability as numerator (Accept $\frac{their\,p}{0.52}$ or $\frac{their\,p}{0.22+0.245+0.25\times their\,0.22}$)
	= 0.471 or $\frac{49}{104}$	A1	
		3	

139. 9709_s21_ms_52 Q: 6

Question	Answer	Marks	Guidance
(a)	$\frac{8!}{2!3!}$	M1	$\frac{8!}{k!\times m!}$ k = 1 or 2, m = 1 or 3, not k = m = 1 no additional terms
	3360	A1	
		2	

Question	Answer	Marks	Guidance
(b)	**Method 1** Arrangements Rs at ends – Arrangements Rs at ends and Os together		
	[Os not together $=]\ \frac{6!}{3!} - 4!$	M1	$\frac{6!}{k!} - m$, $1 \leqslant k \leqslant 3$, m an integer, condone $2\times\left(\frac{6!}{k!}\right) - m$.
		M1	$w - 4!$ or $w - 24$, w an integer Condone $w - 2 \times 4!$
	96	A1	
	Method 2 identified scenarios R _ _ _ R, Arrangement No Os together + 2Os and a single O		
	${}^4C_3 \times 3! + {}^4C_2 \times 2 \times 3!$	M1	${}^4C_3 \times 3! + r$ or $4\times 3! + r$ or ${}^4P_3 \times 3! + r$, r an integer. Condone $2 \times {}^4C_3 \times 3! + r$. $2 \times 4\times 3! + r$ or $2 \times {}^4P_3 \times 3! + r$.
		M1	$q + {}^4C_2 \times 3! \times k$ or $q + {}^4P_2 \times 3! \times k$, $k = 1,2$, q an integer
	[24 + 72 =] 96	A1	
		3	
(c)	**Method 1** Identified scenarios		
	OORR ${}^3C_2 \times {}^2C_2 \times \left[{}^3C_0\right] = 3\times1 = 3$ ORR_ ${}^3C_1 \times {}^2C_2 \times {}^3C_1 = 3\times1\times3 = 9$ OOR_ ${}^3C_2 \times {}^2C_1 \times {}^3C_1 = 3\times2\times3 = 18$ OR_ _ ${}^3C_1 \times {}^2C_1 \times {}^3C_2 = 3\times2\times3 = 18$ OOOR ${}^3C_3 \times {}^2C_1 \times \left[{}^3C_0\right] = 1\times2 = 2$	B1	Outcomes for 2 identifiable scenarios correct, accept unsimplified.
		M1	Add 4 or 5 identified correct scenarios only values, no additional incorrect scenarios, no repeated scenarios, accept unsimplified, condone use of permutations.
	Total 50	A1	All correct and added
	Probability $= \frac{50}{{}^8C_4}$	M1	$\frac{\textit{their}'50'}{{}^8C_4}$, accept numerator unevaluated

Question	Answer	Marks	Guidance
(c) cont'd	$\frac{50}{70}$ or 0.714	A1	
	Method 2 Identified outcomes		
	ORTM ${}^3C_1 \times {}^2C_1 = 6$ ORTW ${}^3C_1 \times {}^2C_1 = 6$ ORMW ${}^3C_1 \times {}^2C_1 = 6$ ORRM ${}^3C_1 \times {}^2C_2 = 3$ ORRW ${}^3C_1 \times {}^2C_2 = 3$ ORRT ${}^3C_1 \times {}^2C_2 = 3$ OROR ${}^3C_2 \times {}^2C_2 = 3$ OROT ${}^3C_2 \times {}^2C_1 = 6$ OROM ${}^3C_2 \times {}^2C_1 = 6$ OROW ${}^3C_2 \times {}^2C_1 = 6$ OROO ${}^3C_3 \times {}^2C_1 = 2$	B1	Outcomes for 5 identifiable scenarios correct, accept unsimplified.
		M1	Add 9, 10 or 11 identified correct scenarios only values, no additional incorrect scenarios, no repeated scenarios, accept unsimplified, condone use of permutations.
	Total 50	A1	All correct and added
	Probability $= \frac{50}{{}^8C_4}$	M1	$\frac{\textit{their}'50'}{{}^8C_4}$, accept numerator unevaluated.
	$\frac{50}{70}$ or 0.714	A1	
		5	

140. 9709_w21_ms_51 Q: 3

Question	Answer	Marks	Guidance
	$\left[\mathrm{P}(T\|B')=\dfrac{\mathrm{P}(T\cap B')}{\mathrm{P}(B')}\right]$ $\mathrm{P}(B')=0.45\times0.7+0.35\times0.4+0.2\times1$ $\left[=0.655,\dfrac{131}{200}\right]$	M1	$0.45\times a+0.35\times b+0.2[\times1], a=0.7, 0.3b=0.4, 0.6$, seen anywhere.
		A1	Correct, accept unsimplified.
	$\mathrm{P}(T\cap B')=0.35\times0.4$ [= 0.14, $\dfrac{7}{50}$]	M1	Seen as numerator or denominator of a fraction.
	$\mathrm{P}(T\mid B')=\dfrac{their\,0.14}{their\,0.655}$	M1	Values substituted into conditional probability formula correctly. Accept unsimplified. Denominator sum of 3 two-factor probabilities (condone omission of 1 from final factor). If clearly identified, condone from incomplete denominator.
	0.214, $\dfrac{28}{131}$	A1	If 0 marks awarded, **SC B1** 0.214 WWW.
		5	

141. 9709_w21_ms_51 Q: 5

Question	Answer	Marks	Guidance
(a)	[8! =] 40 320	B1	Evaluated, exact value only.
		1	
(b)	**Method 1** [^ ^ ^ R ^ ^ S ^ ^]		
	$7!\times{}^{8}C_{2}\times2$	M1	$7!\times k$ seen, k an integer > 1.
		M1	$m\times n(n-1)$ or $m\times{}^{n}C_{2}$ or $m\times{}^{n}P_{2}$, n = 7, 8 or 9, m an integer > 1.
	282 240	A1	Exact value only. **SC B1** for final answer 282 240 WWW.
	Method 2 [Total number of arrangements – Arrangements with R & S together]		
	9! – 8! × 2	M1	9! – k, k an integer < 362 880 .
		M1	m – 8! × n, m an integer > 40 320, n = 1,2.
	282 240	A1	Exact value only. **SC B1** for final answer 282 240 WWW.
		3	
(c)	${}^{9}C_{5}$ [× ${}^{4}C_{4}$]	M1	${}^{9}C_{x}$ [× ${}^{9-x}C_{9-x}$,] x = 4, 5. Condone × 1 for ${}^{9-x}C_{9-x}$. Condone use of P.
	126	A1	WWW
		2	

Question	Answer	Marks	Guidance
(d)	[Number of ways with Raman and Sanjay together on back row =] ${}^{7}C_{3}$ [Number of ways with Raman and Sanjay together on front row =] ${}^{7}C_{2}$	M1	${}^{7}C_{x}$ seen, x = 3 or 2.
	[Total =] 35 + 21	M1	Summing two correct scenarios.
	56	A1	Evaluated – may be seen used in probability. If **M0** scored, **SC B1** for 56 WWW.
	Probability = $\dfrac{their\,56}{their(c)}=\dfrac{56}{126},\dfrac{4}{9}$, 0.444	B1 FT	FT *their* 56 from adding 2 or more scenarios in numerator and *their* **(c)** or correct as denominator.
		4	

142. 9709_w21_ms_52 Q: 1

Question	Answer	Marks	Guidance
(a)	$\frac{82}{180}, \frac{41}{90}$, 0.456	B1	
		1	
(b)	$\left[\mathrm{P(M\|D)} = \frac{P(M \cap D)}{P(D)}\right] = \frac{\frac{11}{180}}{\frac{20}{180}+\frac{11}{180}}$ or $\frac{0.6011}{0.1722}$	M1	*Their* identified $\frac{P(M \cap D)}{P(D)}$ *or* from data table $\frac{11}{20+11}$, accept unsimplified, condone × 180.
	$\frac{11}{31}$, 0.355	A1	Final answer.
		2	

Question	Answer	Marks	Guidance
(c)	$\mathrm{P(F)} = \frac{100}{180}, \frac{5}{9}, 0.5556$ OE $\mathrm{P(G)} = \frac{82}{180}, \frac{41}{90} 0.4556$ OE $\mathrm{P(F \cap G)} = \frac{38}{180}, \frac{19}{90}, 0.2111$ OE $\mathrm{P(F)} \times \mathrm{P(G)} = \frac{100}{180} \times \frac{82}{180} = \frac{41}{162}, 0.2531$ OE $\left[\neq \frac{38}{180}\right]$ Not independent	**M1**	*Their* identified P(F) × *their* identified P(G) or correct seen, can be unsimplified.
		A1	$\frac{41}{162}, \frac{38}{180}, \mathrm{P(F \cap G)}$ and $\mathrm{P(F)} \times \mathrm{P(G)}$ seen with correct conclusion, WWW. Values and labels must be seen.
	Alternative method for question 1(c)		
	$\mathrm{P(F \cap G)} = \frac{38}{180}, \frac{19}{90}, 0.2111$ OE $\mathrm{P(G)} = \frac{82}{180}, \frac{41}{90}, 0.4556$ OE $\mathrm{P(F\|G)} = \frac{\frac{38}{180}}{\frac{82}{180}} = \frac{19}{41}, 0.4634$ OE $\neq \mathrm{P(F)} = \frac{100}{180}, \frac{5}{9}, 0.5556$ OE Not independent	**M1**	P(F|G) (OE) unsimplified with *their* identified probs or correct
		A1	$\frac{19}{41}, \frac{100}{180}, \mathrm{P(F \cap G)}$ and $\mathrm{P(F\|G)}$ seen with correct conclusion WWW. Values and labels must be seen.
		2	

143. 9709_w21_ms_53 Q: 5

Question	Answer	Marks	Guidance
(a)	${}^5\mathrm{P}_2 \times {}^7\mathrm{P}_4$ **or** $5 \times 4 \times 7 \times 6 \times 5 \times 4$	**M1**	${}^5\mathrm{P}_x \times {}^7\mathrm{P}_y$, $1 \leqslant x \leqslant 4$, $1 \leqslant y \leqslant 6$
	16 800	**A1**	
		2	

Question	Answer	Marks	Guidance
5(b)	**Method 1** [Identify scenarios]		
	With A and no 5: $8 \times {}^6P_4$ or $(1 \times 4 \times 6 \times 5 \times 4 \times 3) \times 2$ or $4C1 \times 2! \times 6P4 = 2880$	**M1**	One number of ways correct, accept unsimplified.
	With 5 and no A: ${}^4P_2 \times 4 \times {}^6P_3$ or $(4 \times 3 \times 1 \times 6 \times 5 \times 4) \times 4$ or $4P2 \times 6C3 \times 4! = 5760$ With A and 5: $8 \times 4 \times {}^6P_3$ or $(4 \times 1 \times 1 \times 6 \times 5 \times 4) \times 8$ or $4C1 \times 2! \times 6C3 \times 4! = 3840$	**M1**	Add 2 or 3 identified correct scenarios only, accept unsimplified.
	[Total =] 12 480	**A1**	CAO
	Method 2 [total number of codes – number of codes with no A or 5]		
	No A or 5 : $(4 \times 3) \times (6 \times 5 \times 4 \times 3) = 4320$	**M1**	${}^4P_2 \times {}^6P_4$ or ${}^4C_2 \times {}^6C_4$ seen, accept unsimplified.
	Required number = *their* **(a)** – *their* 4320	**M1**	*Their* **5(a)** (or correct) – *their* (No A or 5) value.
	12 480	**A1**	
	Method 3 [subtracting **double counting**]		
	With A ${}^4P_1 \times {}^7P_4 \times 2$ or ${}^4C_1 \times 2 \times {}^7C_4 \times 4! = 6720$ With 5 ${}^5P_2 \times {}^6P_3 \times 4$ or ${}^5C_2 \times 2 \times {}^6C_3 \times 4! = 9600$ With A and 5 = ${}^4P_1 \times {}^6P_3 \times 8$or $4C1 \times 2! \times 6C3 \times 4! \times 8 = 3840$	**M1**	One outcome correct, accept unsimplified.
	Required number = 6720 + 9600 – 3840	**M1**	Adding 'with a' to 'with 5' and subtracting 'A and 5'.
	12 480	**A1**	CAO
		3	

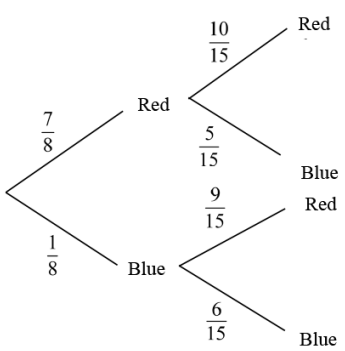
Question	Answer	Marks	Guidance
(c)	**Method 1** – number of successful codes divided by total		
	$(1 \times) 3 \times {}^5P_2$	**M1**	$3 \times {}^5P_n$, $n = 2, 3$. Condone $3 \times {}^5C_2$, no + or –.
	Probability = $\frac{\textit{their}\ 3 \times 5P2}{\textit{their}\ 16\,800}$	**M1**	Probability = $\frac{\textit{their}\ 60}{\textit{their}\ 16\,800}$.
	$\frac{1}{280}$, 0.00357	**A1**	
	Method 2 – product of probabilities of each part of code		
	$\frac{1}{5} \times \frac{1}{4} \times \frac{1}{7} \times \frac{3}{6}\left(\times \frac{5}{5} \times \frac{4}{4}\right)$ or $\frac{1}{5} \times \frac{1}{4} \times \frac{3 \times 5P2}{7P4}$	**M1**	$\frac{1}{5} \times \frac{1}{4} \times k$ where $0 < k < 1$ for considering letters.
		M1	$t \times \frac{1}{7} \times \frac{3}{6}$ or $t \times \frac{3 \times 5P2}{7P4}$ where $0 < t < 1$.
	$\frac{1}{280}$	**A1**	CAO
		3	

144. 9709_w21_ms_53 Q: 7

Question	Answer	Marks	Guidance
(a)	Probabilities: $\frac{x+1}{x+10}, \frac{9}{x+10}, \frac{x}{x+10}, \frac{10}{x+10}$	**B1**	One probability correct in correct position.
		B1	Another probability correct in correct position.
		B1	Other two probabilities correct in correct positions.
		3	
(b)	$\frac{4}{10} \times \textit{their}\ \frac{10}{x+10}$	**M1**	Method consistent with *their* tree diagram.
	$\frac{4}{x+10}$	**A1**	AG
		2	

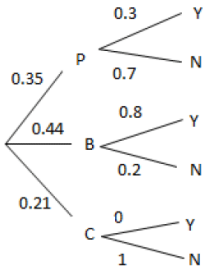
Question	Answer	Marks	Guidance
(c)	$\frac{4}{x+10}=\frac{1}{6}$ $x+10=24, \quad x=14$	**B1**	Find value of x. Can be implied by correct probabilities in calculation.
	P(ARed|BRed) = P(ARed ∩ BRed) ÷ P(BRed) $\frac{\frac{6}{10}\times their\frac{x+1}{x+10}}{\frac{6}{10}\times their\frac{x+1}{x+10}+\frac{4}{10}\times their\frac{x}{x+10}}=\frac{\frac{6}{10}\times\frac{15}{24}}{\frac{6}{10}\times\frac{15}{24}+\frac{4}{10}\times\frac{14}{24}}=\frac{\frac{3}{8}}{\frac{73}{120}}$	**B1 FT**	$\frac{6}{10}\times their\frac{x+1}{x+10}$ as numerator or denominator of fraction.
		M1	$\frac{6}{10}\times their\frac{x+1}{x+10}+\frac{4}{10}\times their\frac{x}{x+10}$ seen anywhere.
		A1 FT	Seen as denominator of fraction.
	$\frac{45}{73}$, 0.616[4…]	**A1**	If B0 M0: **SC B1** for $\frac{\frac{3}{8}}{\frac{73}{120}}$ or $\frac{0.375}{0.6083}$ **SC B1** $\frac{45}{73}$ or 0.616.
		5	

145. 9709_m20_ms_52 Q: 6

Question	Answer	Marks	Guidance
(a)	Box A Box B  	**B1**	Both correct probs, box A
		B1	2 probs correct for box B
		B1	All correct probs for box B
		3	
(b)	$\frac{7}{8}\times\frac{5}{15}+\frac{1}{8}\times\frac{9}{15}$	**M1**	Two 2 factor terms added, correct or FT *their* **6(a)**.
	$=\frac{44}{120}\left[\frac{11}{30}\text{ or }0.367\right]$	**A1**	OE
		2	

Question	Answer	Marks	Guidance
(c)	$\text{P(A blue } \mid \text{B blue)} = \frac{\text{P(A blue} \cap \text{B blue)}}{\text{P(B blue)}}$ $= \frac{\frac{1}{8}\times\frac{6}{15}}{\frac{7}{8}\times\frac{5}{15}+\frac{1}{8}\times\frac{6}{15}} = \frac{\frac{1}{20}}{\frac{41}{120}}$	M1	*their* $\frac{1}{8}\times\frac{6}{15}$ seen as numerator or denom of fraction
		M1	*their* $\frac{7}{8}\times\frac{5}{15}+\frac{1}{8}\times\frac{6}{15}$ seen
		M1	*their* $\frac{7}{8}\times\frac{5}{15}+\frac{1}{8}\times\frac{6}{15}$ seen as denominator
	$= \frac{6}{41}$ or 0.146	A1	
		4	

146. 9709_s20_ms_51 Q: 5

Question	Answer	Marks
(a)		
	Fully correct labelled tree for method of transport with correct probabilities.	B1
	Fully correct labelled branches with correct probabilities for lateness with either 1 branch after W or 2 branches with the prob 0	B1
		2
(b)	$0.35\times0.3+0.44\times0.8\,(+0)$	M1
	0.457	A1
		2

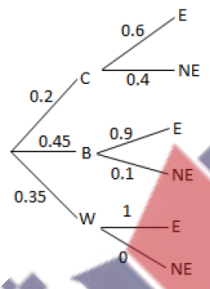
Question	Answer	Marks
(c)	$\text{P(not B} \mid \text{not fruit)} = \frac{\text{P(B'} \cap \text{F')}}{\text{P(F')}}$	M1
	$\frac{0.35\times0.7+0.21\times1}{1-\textit{their}\,(\textbf{b})}$	M1
	$\frac{0.455}{0.543}$ (**M1** for 1 – *their* **(b)** or summing three appropriate 2-factor probabilities, correct or consistent with *their* tree diagram as denominator)	M1
	0.838 or $\frac{455}{543}$	A1
		4

147. 9709_s20_ms_52 Q: 2

Question	Answer	Marks
(a)	$\frac{56}{500}$ or $\frac{14}{125}$ or 0.112	B1
		1
(b)	$P(D\|S)=\frac{P(D\cap S)}{P(S)}=\frac{120}{280}$	M1
	$\frac{120}{280}$ or $\frac{3}{7}$	A1
		2

Question	Answer	Marks
(c)	P(hockey) $=\frac{220}{500}=0.44$ P(Amos or Benn) $=\frac{242}{500}=0.484$ P(hockey $\cap$ A or B) $=\frac{104}{500}=0.208$ P(H) $\times$ P(A U B) = P(H $\cap$ (A U B)) if independent	M1
	$\frac{220}{500}\times\frac{242}{500}=\frac{1331}{6250}$ so not independent	A1
		2

148. 9709_s20_ms_53 Q: 1

Question	Answer	Marks
(a)	 Fully correct labelled tree for method of transport with correct probabilities.	B1
	Fully correct labelled branches with correct probabilities for lateness with either 1 branch after W or 2 branches with the probability 0.	B1
		2
(b)	$P(C\|E)=\frac{P(C\cap E)}{P(E)}=\frac{0.2\times0.6}{0.2\times0.6+0.45\times0.1+0.35\times1}$	M1
	Summing three appropriate 2-factor probabilities	M1
	$\frac{0.12}{0.515}$	A1
	0.233 or $\frac{12}{515}$	A1
		4

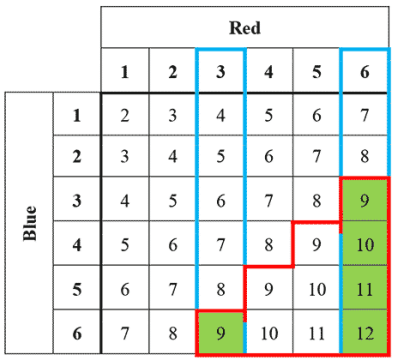
149. 9709_s20_ms_53 Q: 7

Question	Answer	Marks
(a)	$\frac{9!}{2!2!} = 90\,720$	B1
		1
(b)	$\frac{6!}{2!}$	M1
	360	A1
		2

Question	Answer	Marks
(c)	2 Es together $= \frac{8!}{2!} (= 20160)$	M1
	Es not together $= 90720 - 20160 = 70560$	M1
	Probability $= \frac{70560}{90720}$	M1
	$\frac{7}{9}$ or 0.778	A1
	Alternative method for question 7(c)	
	_ ^ _ ^ _ ^ _ ^ _ ^ _ ^ _ ^ _ $\frac{7!}{2!} \times \frac{8 \times 7}{2} = 70560$	
	7! × k in numerator, k integer ⩾ 1, denominator ⩾ 1	M1
	Multiplying by $^8C_2$ OE	M1
	Probability $= \frac{70560}{90720}$	M1
	$\frac{7}{9}$ or 0.778	A1
		4

Question	Answer	Marks
(d)	Scenarios are: E L _ _ _ $^5C_3$ 10 E E L _ _ $^5C_2$ 10 E _ _ _ _ $^5C_4$ 5 E E _ _ _ $^5C_3$ 10	M1
	Summing the number of ways for 3 or 4 correct scenarios	M1
	Total = 35	A1
		3

150. 9709_w20_ms_51 Q: 1

Question	Answer	Marks	Guidance
(a)		M1	Complete outcome space or or listing A and B outcomes or listing A∩B outcomes
	$P(A \cap B) = \frac{5}{36}$	A1	With evidence
		2	

Question	Answer	Marks	Guidance
(b)	$P(A) \times P(B) = \frac{1}{3} \times \frac{10}{36}$	M1	*Their* $\frac{1}{3} \times$ *their* $\frac{10}{36}$ seen
	$\frac{5}{54} \neq \frac{5}{36}$ so not independent	A1	$\frac{5}{54}, \frac{5}{36}$, P(A) × P(B) and P(A∩*B*) seen in workings and correct conclusion stated Condone $\frac{5}{36}$ being stated in **(a)**
	Alternative method for question 1(b)		
	$P(B\|A) = P(B)$ $P(B\|A) = \frac{P(A \cap B)}{P(A)} = \frac{\frac{5}{36}}{\frac{1}{3}}$	M1	OE, $\frac{\textit{their}\ 1(a)}{\textit{their}\ P(A)}$ seen
	$\frac{5}{12} \neq \frac{5}{18}$ so not independent	A1	P(A|B), P(B), $\frac{5}{12}, \frac{5}{18}$ seen in workings and correct conclusion stated Condone $\frac{5}{18} \equiv \frac{10}{36}$ being identified in **(a)**
		2	

151. 9709_w20_ms_51 Q: 2

Question	Answer	Marks	Guidance
(a)	$0{\cdot}6 \times 0{\cdot}7 + 0{\cdot}4(1-x) = 0{\cdot}58$ $\equiv 0{\cdot}42 + 0{\cdot}4(1-x) = 0{\cdot}58$	M1	Equation of form $0{\cdot}6 \times a + 0{\cdot}4 \times b = 0{\cdot}58$; $a = 0{\cdot}3, 0{\cdot}7$, b = x, $(1-x)$
		B1	Single correct product seen, condone 0·42, in an equation of appropriate form
	$x = 0{\cdot}6$	A1	
	Alternative method for question 2(a)		
	$0{\cdot}6 \times 0{\cdot}3 + 0{\cdot}4x = 0{\cdot}42$ $\equiv 0{\cdot}18 + 0{\cdot}4x = 0{\cdot}42$	M1	Equation of form 0·6 x a + 0·4 x b = 0·42; $a = 0{\cdot}3, 0{\cdot}7$, b = x, $(1-x)$
		B1	Single correct product seen, condone 0·18, in an equation of appropriate form
	$x = 0{\cdot}6$	A1	
		3	
(b)	$(0.6\times0.3)^2$	M1	$(a \times b)^2$, $a = 0{\cdot}6, 0{\cdot}4$ and $b = 0{\cdot}7, 0{\cdot}3, x, (1-x)$ or $0{\cdot}18^2$, alone.
	0.0324	A1	
		2	

152. 9709_w20_ms_51 Q: 7

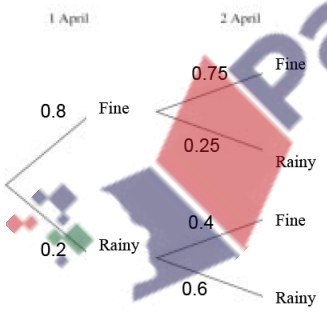
(a)	$\frac{8!}{2!}$	M1	$\frac{8!}{k} \equiv \frac{7\times8}{k}$, where $k \in \mathbb{N}$, $\frac{a!}{2(!)}$, where $a \in \mathbb{N}$
	20160	A1	
		2	

Question	Answer	Marks	Guidance
(b)	Total number of ways: $\frac{10!}{2!3!}$ $(=302\,400)$ (A)	B1	Accept unsimplified
	With Ps together: $\frac{9!}{3!}$ (= 60 480) (B)	B1	Accept unsimplified
	With Ps not together: 302 400 – 60 480	M1	$\frac{10!}{m} - \frac{9!}{n}$, m, n integers or (A) – (B) if clearly identified
	241 920	A1	
	Alternative method for question 7(b)		
	$\frac{8!}{3!}$	B1	$k \times 8!$ in numerator, k a positive integer, no ±
		B1	$m \times 3!$ in denominator, m a positive integer, no ±
	$\times\frac{9\times8}{2}$	M1	*Their* $\frac{8!}{3!}$ multiplied by ${}^9C_2$ or ${}^9P_2$ no additional terms
	241 920	A1	Exact value, WWW
		4	

Question	Answer	Marks	Guidance
(c)	Probability = $\frac{\text{Number of ways Es at beginning and end}}{\text{Total number of ways}}$ Probability = $\frac{\frac{8!}{2!}}{\frac{10!}{2!\times 3!}} = \frac{20160}{302400}$	M1	$\frac{\left(\frac{8!}{k!}\right)}{\frac{10!}{k!l!}}$ $1 \leqslant k, l \in \mathbb{N} \leqslant 3$, FT denominator from **7(b)** or correct
	$\frac{1}{15}$, 0·0667	A1	
	Alternative method for question 7(c)		
	Probability = $\frac{3}{10}\times\frac{2}{9}$	M1	$\frac{a}{10}\times\frac{a-1}{9}\ a=3,2$
	$\frac{1}{15}$, 0·0667	A1	
	Alternative method for question 7(c)		
	Probability = $\frac{1}{10}\times\frac{1}{9}\times 3!$	M1	$\frac{1}{10}\times\frac{1}{9}\times m!, m=3,2$
	$\frac{1}{15}$, 0.0667	A1	
		2	

Question	Answer	Marks	Guidance
(d)	Scenarios: P E E E $^5C_0 = 1$ P E E _ $^5C_1 = 5$ P E _ _ $^5C_2 = 10$ P _ _ _ $^5C_3 = 10$	M1	^5C_x seen alone, $1 \leqslant x \leqslant 4$
		M1	Summing the number of ways for 3 or 4 correct scenarios (can be unsimplified), no incorrect scenarios
	Total = 26	A1	
		3	

153. 9709_w20_ms_52 Q: 4

Question	Answer	Marks	Guidance
(a)		B1	All probabilities correct, may be on branch or next to 'Fine/Rainy' Ignore additional branches.
		1	
(b)	0·8 × 0·75 + 0·2 × 0·4 (= 0·6 + 0·08)	M1	Correct or FT from *their* diagram unsimplified, all probabilities $0 < p < 1$. Partial evaluation only sufficient when correct. Accept working in **4(b)** or by the tree diagram.
	0.68, $\frac{17}{25}$	A1	From supporting working
		2	

Question	Answer	Marks	Guidance
(c)	$0{\cdot}8 \times 0{\cdot}75 \times 0{\cdot}25 + 0{\cdot}8 \times 0{\cdot}25 \times 0{\cdot}6$	**M1**	$a \times b \times c + a \times 1\text{-}b \times d$, $0 < c$, $d \leqslant 1$, a, b consistent with *their* tree diagram or correct, no additional terms
	$0{\cdot}15 + 0{\cdot}12$	**A1**	At least one term correct, accept unsimplified
	0.27	**A1**	Final answer
		3	
(d)	P(Y) = *their* **(c)** + $0{\cdot}2 \times 0{\cdot}4 \times 0{\cdot}25 + 0{\cdot}2 \times 0{\cdot}6 \times 0{\cdot}6$ (= $0{\cdot}362$)	**B1 FT**	*their* **(c)** + $e \times f \times g + e \times (1\text{-}f) \times h$, $0 < g$, $h \leqslant 1$, e, f consistent with *their* tree diagram, or correct
	$\text{P(X\|Y)} = \frac{\textit{their}\,(\text{c})}{\textit{their}\,\text{P(Y)}} = \frac{0.27}{0.362}$	**M1**	*their* **4(c)** (or correct)/*their* previously calculated and identified P(Y) or a denominator involving 3 or 4 3-factor probability terms consistent with *their* tree diagram & third factor $0 < p < 1$
	$0{\cdot}746$, $\frac{373}{500}$ or $\frac{135}{181}$	**A1**	(0.7458…)
		3	

154. 9709_w20_ms_52 Q: 6

Question	Answer	Marks	Guidance
(a)	${}^9C_6$ (× ${}^3C_3$)	**M1**	${}^9C_k \times n$, $k = 6, 3$, $n = 1, 2$ oe Condone ${}^9C_6 + {}^3C_3$, ${}^9P_6 \times {}^3P_3$
	84	**A1**	Accept unevaluated.
		2	
(b)	Number with 3 Baker children = ${}^6C_2$ or 15	**B1**	Correct seen anywhere, not multiplied or added
	Total no of selections = ${}^9C_5$ or 126 $\text{Probability} = \frac{\text{number of selections with3 Baker children}}{\text{total number of selections}}$	**M1**	Seen as denominator of fraction
	$\frac{15}{126}$, $0{\cdot}119$	**A1**	OE, e.g. $\frac{5}{42}$
	Alternative method for question 6(b)		
	$\frac{3}{9} \times \frac{2}{8} \times \frac{1}{7}\left(\times\frac{6}{6}\right)\left(\times\frac{5}{5}\right) \times {}^5C_3$	**B1**	${}^5C_3$ (OE) or 10 seen anywhere, multiplied by fractions only, not added
		M1	$\frac{3}{9} \times \frac{2}{8} \times \frac{1}{7}\left(\times\frac{6}{6}\right)\left(\times\frac{5}{5}\right) \times k$, $1 \leqslant k$, k integer
	$\frac{15}{126}$, $0{\cdot}119$	**A1**	OE, e.g. $\frac{5}{42}$
		3	

Question	Answer	Marks	Guidance
(c)	[Total no of arrangements = 9!] [Arrangements with men together = 8! × 2] Not together: 9! –	M1	9! – k or 362880 – k, k an integer<362 880
	8! × 2	B1	8! × 2(!) or 80 640 seen anywhere
	282 240	A1	Exact value
	Alternative method for question 6(c)		
	7! × 8 × 7	B1	7! × k, k positive integer > 1
		M1	m × 8 × 7, m × ${}^{8}P_{2}$, m × ${}^{8}C_{2}$ m positive integer > 1
	282 240	A1	Exact value
		3	
(d)	7! × 2 × 7	M1	7! × k, k positive integer > 1 If 7! not seen, condone 7 × 6 × 5 × 4 × 3 × 2 × (1) × k *or* 7 × 6! × k only
		M1	m × 2 × 7, m positive integer > 1
	70 560	A1	
		3	

155. 9709_w20_ms_53 Q: 5

Question	Answer	Marks	Guidance
(a)	Total number of ways = $\frac{8!}{3!2!}$ (= 3360)	B1	Correct unsimplified expression for total number of ways
	Number of ways with V and E in correct positions = $\frac{6!}{2\times 2!}$ (= 180)	B1	$\frac{6!}{2\times 2!}$ alone or as numerator in an attempt to find the number of ways with V and E in correct positions. No ×,±
	Probability = $\frac{180}{3360}$ $\left(=\frac{3}{56}\right)$ or 0.0536	B1 FT	Final answer from *their* $\frac{6!}{2\times 2!}$ divided by *their* total number of ways
	Alternative method for question 5(a)		
	$\frac{1}{8}\times\frac{3}{7}$	M1	$\frac{a}{8}\times\frac{b}{7}$ seen, no other terms (correct denominators)
		M1	$\frac{1}{c}\times\frac{3}{d}$ seen, no other terms (correct numerators)
	$\frac{3}{56}$ or 0.0536	A1	
		3	

Question	Answer	Marks	Guidance
(b)	Rs together and Es together: 5! (120)	**B1**	Alone or as numerator of probability to represent the number of ways with Rs and Es together, no ×, +, –
	Es together: $\frac{6!}{2!}(=360)$	**B1**	Alone or as denominator of probability to represent the number of ways with Es together, no ×, + or –
	Probability = $\frac{\frac{5!}{6!}}{2!}$	**M1**	$\frac{\textit{their}\,5!}{\textit{their}\,\frac{6!}{2!}}$ seen
	$\frac{1}{3}$	**A1**	OE
	Alternative method for question 5(b)		
	P(Rs together and Es together): $\frac{5!}{\textit{their}\text{ total number of ways}}\left(=\frac{1}{28}\right)$	**B1**	
	P(Es together): $\frac{\frac{6!}{2!}}{\textit{their}\text{ total number of ways}}\left(=\frac{3}{28}\right)$	**B1**	Alone or as numerator of probability to represent the P(Rs and Es together), no ×, +, –
	Probability = $\frac{\frac{1}{28}}{\frac{3}{28}}$	**M1**	Alone or as denominator of probability to represent the P(Es together), no ×, + or –
	$\frac{1}{3}$	**A1**	OE, $\frac{\textit{their}\,\frac{1}{28}}{\textit{their}\,\frac{3}{28}}$ seen
		4	

156. 9709_m19_ms_62 Q: 1

Question	Answer	Marks	Guidance
(i)	0.6 × 0.2 + 0.4 × 0.32	**M1**	Addition of 2 two-factor terms $0.6 \times a + 0.4 \times b$
	= 0.248, $\frac{31}{125}$	**A1**	CAO
		2	
(ii)	**Method 1**		
	P(GS|Not Red socks) = $\frac{0.4\times 0.68}{1-(i)}$	**B1**	Correct [unsimplified] numerator seen in fraction
		M1	1 – their **(i)** as denominator in fraction
	= 0.362, $\frac{17}{47}$	**A1**	
	Method 2		
	P(GS|Not Red socks) = $\frac{0.4\times 0.68}{0.6\times 0.8+0.4\times 0.68}$	**B1**	Correct [unsimplified] numerator seen in fraction
		M1	Correct or (their **(i)**)' as denominator in fraction
	= 0.362, $\frac{17}{47}$	**A1**	
		3	

157. 9709_s19_ms_61 Q: 2

Question	Answer	Marks	Guidance
	Jameel: P(plum) = $\frac{5}{8}$, Rosa: P(plum) = $\frac{x}{x+6}$ $\frac{5}{8}\times\frac{x}{x+6}=\frac{1}{4}$	M1	*Their* 2 probabilities for P(plum) multiplied and equated to 1/4
		A1	Correct equation oe
	$(x=)\ 4$	A1	**SC** correct answer with no appropriate equations i.e. common sense **B1**
		3	

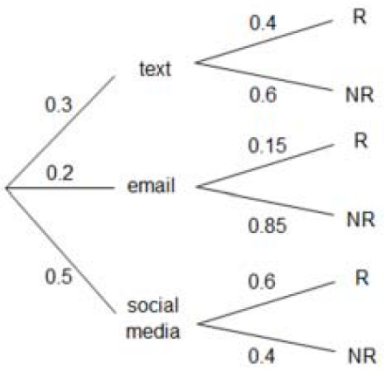
158. 9709_s19_ms_61 Q: 3

Question	Answer	Marks	Guidance
	P(X) = $\frac{3}{36}$ $\left(\frac{1}{12}oe\right)$	B1	
	P(Y) = $\frac{12}{36}$ $\left(\frac{1}{3}oe\right)$	B1	
	P(X∩Y) = $\frac{1}{36}$	M1	Independent method to find P($X\cap Y$) without multiplication, either stated or by listing or circling numbers on a probability space diagram. OR condititional prob with a single fraction numerator
	P(X) × P(Y) = P(X∩Y), independent	A1	Numerical comparison and conclusion, www
		4	

159. 9709_s19_ms_62 Q: 1

Question	Answer	Marks	Guidance
	P(S) = $\frac{1}{2}$	B1	
	P(T) = $\frac{16}{36}$ $\left(\frac{4}{9}\right)$	B1	
	P($S\cap T$) = $\frac{10}{36}$ $\left(\frac{5}{18}\right)$	M1	P($S\cap T$) found by multiplication scores M0 M1 awarded if *their* value is identifiable in their sample space diagram **or** Venn diagram **or** list of terms **or** probability distribution table (oe)
	P(S) P(T) ≠ P($S\cap T$) so not independent	A1	8/36, 10/36 P(S) × P(T) and P($S\cap T$) seen in workings and correct conclusion stated, www
	Alternative method for question 1		
	P(S) = $\frac{1}{2}$	B1	
	P(T) = $\frac{16}{36}$ $\left(\frac{4}{9}\right)$	B1	
	P($S\cap T$) = $\frac{10}{36}$ $\left(\frac{5}{18}\right)$	M1	P($S\cap T$) found by multiplication scores M0 M1 awarded if *their* value is identifiable in their sample space diagram **or** Venn diagram **or** list of terms **or** probability distribution table (oe)
	P(S|T) = $\frac{10}{16}$ or P(T|S) = $\frac{10}{18}$ P(S|T) ≠ P(S) or P(T|S) ≠ P(T) so not independent	A1	**Either** 18/36, 10/16,P(S) and P($S|T$) seen in workings and correct conclusion stated, www **Or** 16/36, 10/18, P(T) and P(T|S) seen in workings and correct conclusion stated, www
		4	

160. 9709_s19_ms_63 Q: 2

Question	Answer	Marks	Guidance
(i)		B1	Fully correct labelled tree with correct probabilities for 'Send'
		B1	Fully correct labelled branches with correct probabilities for the 'reply'
		2	

Question	Answer	Marks	Guidance
(ii)	$P(email \mid NR) = \frac{P(\text{email} \cap \text{NR})}{P(\text{NR})} = \frac{0.2 \times 0.85}{0.3 \times 0.6 + 0.2 \times 0.85 + 0.5 \times 0.4}$	M1	P(email) × P(NR) seen as numerator of a fraction, consistent with *their* tree diagram
	$= \frac{0.17}{0.18 + 0.17 + 0.2} = \frac{0.17}{0.55}$	M1	Summing three appropriate 2-factor probabilities, consistent with *their* tree diagram, seen anywhere 0.55 oe (can be unsimplified) seen as denom of a fraction
	$= 0.309, \frac{17}{55}$	A1	
		A1	Correct answer
		4	

161. 9709_w19_ms_61 Q: 1

Question	Answer	Marks	Guidance
	$0.8 \times 0.6 + 0.2(1-x) = 0.63$	M1	Equation of form $0.8 \times A + 0.2 \times B = C$, A,B involving $1-x$ and 0.6 or 0.4 and C = 0.63 or 0.37
	$0.2x = 0.05$	M1	Correct unsimplified equation
	$x = 0.25$	A1	
	Alternative method for question 1		
	$0.8 \times 0.4 + 0.2x = 1 - 0.63$	M1	Equation of form $0.8 \times A + 0.2 \times B = C$, A,B involving x and 0.6 or 0.4 and C = 0.63 or 0.37
	$0.2x = 0.05$	M1	Correct unsimplified equation
	$x = 0.25$	A1	
		3	

162. 9709_w19_ms_62 Q: 2

Question	Answer	Marks	Guidance
(i)	$0.4x + 0.6 \times 2x = 0.36$ or $0.4(1-x) + 0.6(1-2x) = 0.64$	M1	$0.4a + (1-0.4)b = 0.36$ or 0.64, a,b terms involving x
	$1.6x = 0.36$ $x = 0.225$	A1	Fully justified by algebra AG
		2	

Question	Answer	Marks	Guidance
(ii)	$P(H\|L') = \frac{0.4(1-x)}{1-0.36} = \frac{0.4\times(1-0.225)}{0.64} = \frac{0.4\times0.775}{0.4\times0.775+0.6\times0.55}$	M1	Correct numerical numerator of a fraction. Allow unsimplified.
		M1	Denominator 0.36 or 0.64. Allow unsimplified.
	$\frac{31}{64}$ or 0.484	A1	
		3	

163. 9709_w19_ms_62 Q: 7

Question	Answer	Marks	Guidance
(i)	$6! = 720$	B1	Evaluated
		1	
(ii)	Total no of arrangements: $\frac{9!}{2!3!} = 30240$	B1	Accept unevaluated
	No with Ts together $= \frac{8!}{3!} = 6720$	B1	Accept unevaluated
	With Ts not together: 30 240 – 6720	M1	correct or $\frac{9!}{m} - \frac{8!}{n}, m, n$ integers > 1 or *their* identified total – *their* identified Ts together
	23 520	A1	CAO
	Alternative method for question 7(ii)		
	$\frac{7!}{3!}\times\frac{8\times7}{2}$	B1	$7! \times (k > 0)$ in numerator, cannot be implied by $^7P_2$, etc.
		B1	$3! \times (k > 0)$ in denominator
		M1	$\frac{\textit{their } 7!}{\textit{their } 3!} \times {}^8C_2$ or $^8P_2$
	23 520	A1	CAO
		4	

Question	Answer	Marks	Guidance
(iii)	Number of arrangements $= \frac{7!}{3!}$ Probability $= \frac{\textit{their}\ \frac{7!}{3!}}{\textit{their}\ \frac{9!}{3!2!}} = \frac{840}{30240}$	M1	$\frac{\textit{their}\text{ identified number of arrangements with T at ends}}{\textit{their}\text{ identified total number of arrangements}}$ or $\frac{\frac{7!}{m}}{\frac{9!}{n}} m, n$ integers > 1
	$\frac{1}{36}$ or 0.0278	A1	Final answer
		2	
(iv)	OOT_ _ $^4C_2 = 6$ OOTT_ $^4C_1 = 4$ OOOT_ $^4C_1 = 4$ OOOTT $= 1$	M1	^4C_x seen alone or ^4C_x x $k \geq 1$, k an integer, $0 < x < 4$
		A1	$^4C_2$ x k, $k = 1$ oe or $^4C_1$ x m, $m = 1$ oe alone
		M1	Add 3 or 4 identified correct scenarios only, accept unsimplified
	(Total) = 15	A1	CAO, WWW Only dependent on 2nd M mark
		4	

164. 9709_w19_ms_63 Q: 1

Question	Answer	Marks	Guidance
(i)	$\frac{120}{300} = 0.4$	**B1**	OE
		1	
(ii)	P(male) × P(not piano) = $\frac{160}{300} \times \frac{225}{300}$ $\left(\frac{8}{15} \times \frac{3}{4}\right) = \frac{2}{5}$	**M1**	P(M) × P(P') seen Can be unsimplified but the events must be named in a product
	As P(male ∩ not piano) also = $\frac{120}{300} = \frac{2}{5}$ The events are **Independent**	**A1**	Numerical comparison and correct conclusion
	Alternative method for question 1(ii)		
	P(male ∩ not piano) = $\frac{120}{300}$; P(not piano) = $\frac{225}{300}$	**M1**	P(M|P') or P(P'|M) unsimplified seen with *their* probs with correctly named events
	P(M | not piano) = $\frac{\frac{120}{300}}{\frac{225}{300}} = \frac{120}{225} = \frac{8}{15}$ = P(male) **or** P(not piano | M) = $\frac{\frac{120}{300}}{\frac{160}{300}} = \frac{120}{160} = \frac{3}{4}$ = P(not piano) Therefore the events are **Independent**	**A1**	Numerical comparison with P(M) or P(P') and correct conclusion
		2	

165. 9709_m18_ms_62 Q: 3

Question	Answer	Marks	Guidance
(i)	(10/160 =) 1/16, 0.0625	**B1**	OE
		1	
(ii)	(90/160) = 9/16, 0.5625	**B1**	OE
		1	
(iii)	P(red/hatchback) = P(red hatchback) / P(hatchback) = 40/160 / 90/160	**M1**	Appropriate probabilities in a fraction
	= 4/9	**A1**	OE *Altn method: Direct from table* *M1 for 40/a or b/90, a ≠ 160* *A1 for 40/90 oe*
		2	

Question	Answer	Marks	Guidance
(iv)	*EITHER:* P(red) × P(hatchback) = $\frac{72}{160} \times \frac{90}{160} \neq \frac{40}{160}$	**(M1**	Use correct approach with appropriate probabilities substituted
	Not independent	**A1)**	Numerical comparison and conclusion stated
	OR: P(red/hatchback) = 40/90 and $\frac{40}{90} \neq \frac{72}{160}$	**(M1**	Use correct approach with appropriate probabilities substituted
	Not independent	**A1)**	Numerical comparison and conclusion stated
		2	

166. 9709_s18_ms_62 Q: 2

Question	Answer	Marks	Guidance
(i)	**Method 1** $P(M \cap H) = \frac{3}{4} \times \frac{3}{5} = \frac{9}{20}$ (0.45)	**B1**	Seen, accept unsimplified
	$P(F \text{ or } M \cap H) = \frac{1}{4} + \frac{9}{20} = \frac{14}{20}$	**M1**	Numerical attempt at $P(F) + P(M \cap H)$
		A1	Correct unsimplified expression
	$= \frac{7}{10}$ (0.7) OE	**A1**	Correct final answer
	Method 2 $P(M \cap H') = \frac{3}{4} \times \frac{2}{5} = \frac{6}{20}$ (0.3)	**B1**	Seen, accept unsimplified
	$P(F \text{ or } M \cap H) = 1 - P(M \cap H')$	**M1**	Numerical attempt at $1 - P(M \cap H')$
	$= 1 - \frac{3}{4} \times \frac{2}{5}$	**A1**	Correct unsimplified expression
	$= \frac{7}{10}$ (0.7) OE	**A1**	Correct final answer

Question	Answer	Marks	Guidance
(i)	**Method 3** $P(F \cap H' \text{ or } H) = \frac{1}{4} \times \frac{1}{5} + \frac{1}{4} \times \frac{4}{5} + \frac{3}{4} \times \frac{3}{5}$	**B1**	$\frac{3}{4} \times \frac{3}{5}$ ($\frac{9}{20}$) **or** $\frac{1}{4} \times \frac{4}{5}$ ($\frac{4}{20}$) **or** $\frac{3}{4} \times \frac{3}{5} + \frac{1}{4} \times \frac{4}{5}$ ($\frac{13}{20}$) seen
	$= \frac{1}{20} + \frac{4}{20} + \frac{9}{20}$	**M1**	Numerical attempt at $P(F \cap H') + P(F \cap H) + P(M \cap H)$
		A1	Correct unsimplified expression
	$= \frac{7}{10}$ (0.7) oe	**A1**	Correct final answer
	Method 4 – Venn diagram style approach $P(F \cup H) = P(F) + P(H) - P(F \cap H)$	**B1**	$\frac{3}{4} \times \frac{3}{5}$ ($\frac{9}{20}$) **or** $\frac{1}{4} \times \frac{4}{5}$ ($\frac{4}{20}$) **or** $\frac{3}{4} \times \frac{3}{5} + \frac{1}{4} \times \frac{4}{5}$ ($\frac{13}{20}$) seen
	$= \frac{1}{4} + \frac{1}{4} \times \frac{4}{5} + \frac{3}{4} \times \frac{3}{5} - \frac{1}{4} \times \frac{4}{5}$	**M1**	Numerical attempt at $P(F) + P(H) - P(F \cap H)$
	$= \frac{1}{4} + \frac{4}{20} + \frac{9}{20} - \frac{4}{20}$	**A1**	Correct unsimplified expression
	$= \frac{7}{10}$ (0.7) oe	**A1**	Correct final answer
		4	

Question	Answer	Marks	Guidance
(ii)	**Method 1** $(P(M) \times P(H) =) \frac{3}{4} \times \textit{their} \frac{13}{20} = \frac{39}{80}$ $(P(M \cap H) =) \frac{3}{4} \times \frac{3}{5} = 0.45$	**M1**	Unsimplified, or better, legitimate numerical attempt at $P(M) \times P(H)$ **and** $P(M \cap H)$ Descriptors $P(M \cap H)$ **and** $P(M) \times P(H)$ seen, correct numerical evaluation and comparison, conclusion stated
	$\frac{39}{80}$ $(0.4875) \neq 0.45$, not independent	**A1**	
	Method 2 $P(M \mid H) = \frac{P(M \cap H)}{P(H)} = \frac{\frac{9}{20}}{\text{their } \frac{13}{20}} = \frac{9}{13}$ $P(M) = \frac{3}{4}$	**M1**	Unsimplified, or better, numerical attempt at $P(H)$ and $P(M \cap H)$, $P(M)$
	$\frac{9}{13} \neq \frac{3}{4}$, not independent	**A1**	Descriptors $P(M \cap H)$, $P(H)$ **and** $P(M)$ **OR** $P(M\|H)$ **and** $P(M)$ seen, numerical evaluation **and** comparison, conclusion stated Any appropriate relationship can be used, the M is awarded for an unsimplified, or better, numerical attempt at the terms required, the A mark requires the correct descriptors, numerical evaluation and comparison and the conclusion
		2	

167. 9709_s18_ms_63 Q: 3

Question	Answer	Marks	Guidance
(i)	$(1-x)$ and 0.45 (or 0.3)	**B1**	Seen, either on tree diagram or elsewhere
	Beginners: $0.7 \times x$ + '0.45' × '$(1-x)$' = 0.5 Or Advanced: '0.3' × x + 0.55 × '$(1-x)$' = 0.5 Or $0.7 \times x$ + '0.45'× '$(1-x)$' = '0.3' × x + 0.55× '$(1-x)$'	**M1**	One of the three correct probability equations
	$x = 0.2$ oe	**A1**	Correct answer
	Total:	**3**	
(ii)	$P(M \mid A) = \frac{P(M \cap A)}{P(A)} = \frac{0.2 \times 0.3}{0.5}$	**M1**	'i' × 0.3 as num or denom of a fraction
		M1	0.5 (or (1 – 'i') × 0.55 + 'i' × 0.3 unsimplified) seen as denom of a fraction
	$= 0.12 \left(\frac{3}{25}\right)$	**A1**	Correct answer
	Total:	**3**	

168. 9709_w18_ms_61 Q: 7

Question	Answer	Marks	Guidance
(i)	52/160 = 13/40, 0.325	**B1**	oe
		1	
(ii)	P(boy) = 96/160: P(Music) = 52/160 P(boy and Music) = 40/160	**M1**	Use of P(B) × P(M) = P(B∩M), appropriate probabilities used
	96/160 × 52/160 ≠ 40/160: Not independent	**A1**	Numerical comparison and conclusion stated
		2	

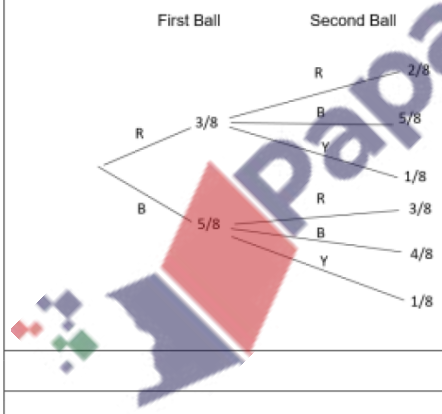
Question	Answer	Marks	Guidance
(iii)	**Method 1**		
	P(not Music/girl) = P(not Music and girl)/P(girl) (27/160) / (64/160)	**M1**	Appropriate probabilities in a fraction
	$= \frac{27}{64}$	**A1**	Correct answer www implies method
	Method 2		
	Direct from table	**M1**	*27/a or b/64, a ≠ 160*
	$\frac{27}{64}$	**A1**	Correct answer www implies method
		2	
(iv)	P(B M) × P(B NM) × P(G NM) **or** P(G M) × P(B NM) ×P(B NM)	**M1**	One scenario identified with 3 probs multiplied
	40/160 × 56/159 × 52/158 **or** 12/160 × 56/159 × 55/158	**A1**	One scenario correct (ignore multiplying factor)
	× 3! × 3!/2!	**B1**	Both multiplying factors correct
	0.17387 0.02759 P = 0.17387 + 0.02759	**M1**	Both cases attempted and added (multiplying factor not required), accept unsimplified
	= 0.201 Note: If score in this part is 0, award SCB1 for $\frac{1}{160}\times\frac{1}{159}\times\frac{1}{158}\times k$, for positive integer k, seen	**A1**	Correct answer, oe

Question	Answer	Marks	Guidance
(iv)	**Method 2**		
	$\frac{\binom{40}{1}\times\binom{56}{1}\times\binom{52}{1}+\binom{12}{1}\times\binom{56}{2}}{\binom{160}{3}}$	**M1**	One scenario identified with 2 or 3 combination multiplied
		A1	One scenario correct
		B1	Denominator correct
	$\frac{116480+18480}{669920}$	**M1**	Both scenarios attempted, and added, seen as a numerator of a fraction
	$\frac{1687}{8374}$	**A1**	Correct answer, oe
		5	

169. 9709_w18_ms_62 Q: 1

Question	Answer	Marks	Guidance
(i)	$\frac{11!}{4!4!2!}$	M1	$\frac{11!}{4!\times k}$ *or* $\frac{11!}{2!\times k}$, k a positive integer
	= 34650	A1	Correct final answer
		2	
(ii)	**Method 1**		
	P(SS) = $\frac{4}{11}\times\frac{3}{10}=\frac{12}{110}$ (= 0.10911)	B1	One of P(SS), P(PP) or P(II) correct, allow unsimplified
	P(PP) = $\frac{2}{11}\times\frac{1}{10}=\frac{2}{110}$ (= 0.01818) P(II) = $\frac{4}{11}\times\frac{3}{10}=\frac{12}{110}$ (= 0.10911) $\frac{4}{11}\times\frac{3}{10}$	M1	Sum of probabilities from 3 appropriate identifiable scenarios (either by labelling or of form $\frac{4}{11}\times\frac{a}{b}+\frac{2}{11}\times\frac{c}{b}+\frac{4}{11}\times\frac{a}{b}$ where a = 4 *or* 3, b = 11 *or* 10, c = 2 *or* 1)
	Total = $\frac{26}{110}=\frac{13}{55}$ oe (0.236)	A1	Correct final answer
	Method 2		
	Total number of selections = $^{11}C_2 = 55$ Selections with 2 Ps = 1	B1	Seen as the denominator of fraction (no extra terms) allow unsimplified
	Selections with 2 Ss = $^4C_2 = 6$ Selections with 2 Is = $^4C_2 = 6$,	M1	Sum of 3 appropriate identifiable scenarios (either by labelling or values, condone use of permutations. May be implied by 2,12,12)
	Total selections with 2 letters the same = 13 Probability of 2 letters the same = $\frac{13}{55}$ oe (0.236)	A1	Correct final answer, without use of permutations
		3	

170. 9709_w18_ms_63 Q: 3

Question	Answer	Marks	Guidance
(i)		B1	Fully correct labelled tree and correct probabilities for 'First Ball'
		B1	Correct probabilities (with corresponding labels) for 'Second Ball'
		2	
(ii)	P(RR) + P(BB) = 3/8 × 2/8 + 5/8 × 4/8 = 3/32 + 5/16	M1	Correct unsimplified expression from their tree diagram, Σp = 1 on each branch
	= 13/32 (0.406)	A1	Correct answer
		2	

Question	Answer	Marks	Guidance
(iii)	P(RB) = 3 / 8 × 5 / 8 = 15/64	**M1**	$P(\text{1st ball red}) \times P(\text{2nd ball blue})$ from their tree diagram seen unsimplified as numerator or denominator of a fraction Allow $\Sigma p \neq 1$ on each branch
	P(B) = 3/8 × 5/8 + 5/8 × 4/8 = 35/64	**M1**	Correct unsimplified expression for P(B) from their tree diagram seen as denominator of a fraction. Allow $\Sigma p \neq 1$ on each branch
	P(R|B) = P(RB) / P(B) = (15/64) ÷ (35/64) = 3/7 (0.429)	**A1**	Correct answer
		3	

171. 9709_w18_ms_63 Q: 4

Question	Answer	Marks	Guidance
(i)	Total number of selections = $^{12}C_7 = 792$	**B1**	Seen as denominator of fraction
	Selections with boy included = $^{11}C_6$ or $^{12}C_7 - {}^{11}C_7 = 462$	**M1**	Correct unsimplified expression for selections with boy included seen as numerator of fraction
	Probability = 462/792 = 7/12 (0.583)	**A1**	Correct answer
	OR		
	prob of boy not included = 11/12 × 10/11 × …. × 5/6= 5/12	**B1**	Correct unsimplified prob
	1 – 5/12	**M1**	Subtracting prob from 1
	= 7/12	**A1**	Correct answer
		3	

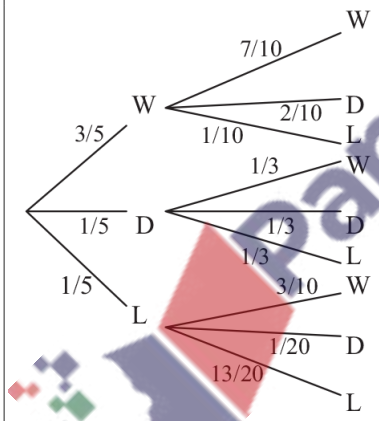
Question	Answer	Marks	Guidance
(ii)	**Method 1**		
	Scenarios are: 2G + 5B: $^4C_2 \times {}^8C_5 = 336$	**B1**	One unsimplified product correct
	3G + 4B: $^4C_3 \times {}^8C_4 = 280$ 4G + 3B: $^4C_4 \times {}^8C_3 = 56$	**M1**	No of selections (products of ^nC_r and ^nP_r) added for 2, 3 and 4 girls with no of girls and no of boys summing to 7
	Total = 672	**A1**	Correct total
	Probability = 672/792 (28/33) (0.848)	**A1ft**	Correct answer – 'total'/('total no of selections' from i)
	Method 2		
	0G + 7B $^4C_0 \times {}^8C_7 = 8$	**B1**	One unsimplified no of selections correct
	1G + 6B $^4C_1 \times {}^8C_6 = 112$ Total = 8 + 112 = 120	**M1**	No of selections (products of ^nC_r and ^nP_r) added for 0 and 1 girls with no of girls and no of boys summing to 7
	($^{12}C_7$ – 120)/792 or 1 – 120/792	**A1**	792 – 120 = 672 or 1 – 120/792
	Probability = 672/792 (28/33) (0.848)	**A1ft**	'672' over '792' from i
	Method 3 (probability)		
	1 – P(0) – P(1) = 1 – (8/12 × 7/11 × …… × 2/6) – (8/12 × … × 3/7 × 4/6 × 7)	**B1**	One correct unsimplified prob for 0 or 1
	= 1 – 1/99 –14/99	**M1**	Subtracting 'P(0)' and 'P(1)' (using products of 7 fractions with denominators from 12 to 6) from 1
		A1	Both probs correct unsimplified
	= 84/99 = 28/33	**A1ft**	1 – 'P(0)' – 'P(1)'

Question	Answer	Marks	Guidance
(ii)	**Method 4 (probability)**		
	P(2) + P(3) + P(4) =	**B1**	One correct unsimplified prob for 2, 3 or 4
	42/99 + 35/99 + 7/99	**M1**	Adding 'P(2)', 'P(3)' and P(4)' (using products of 7 fractions with denominators from 12 to 6)
		A1	Three probs correct unsimplified
	= 84/99 = 28/33	**A1ft**	'P(2)'+ 'P(3)' + 'P(4)'
		4	

172. 9709_s17_ms_61 Q: 2

Question	Answer	Marks	Guidance
	P(R) = 4/36 = 1/9	**M1**	Attempt at P(R) by probability space diag or listing more than half the options, must see a prob, just a list is not enough
	$P(T)=P(O,E)+P(E,O)=1/4+1/4=1/2$ OR $P(R\|T)=1/9$	**M1**	Attempt at P(T) or P(R|T) involving more than half the options
	$P(R\cap T)=P(3,4)+P(4,3)=2/36=1/18$ OR $P(R\|T)=1/9$	**B1**	Value stated, not from P(R) × P(T) e.g. from probability space diagram
	As $P(R)\times P(T)=P(R\cap T)$ OR as $P(R\|T)=P(R)$	**M1**	Comparing product values with $P(R\cap T)$, or comparing P (R|T) with P(R)
	The events are independent.	**A1**	Correct conclusion must have all probs correct
	Total:	**5**	

173. 9709_s17_ms_61 Q: 3

Question	Answer	Marks	Guidance
(i)		**M1**	Correct shape i.e. 3 branches then 3 by 3 branches, labelled and clear annotation Condone omission of lines for first match result providing the probabilities are there.
		A1	All correct probs with fully correct shape and probs either fractions or decimals not 1.5/5 etc.
	Total:	**2**	

Question	Answer	Marks	Guidance
(ii)	$P(L_1 \text{ given } W_2) = \frac{P(L_1 \cap W_2)}{P(W_2)}$	**M1**	Attempt at P(L1∩W2) as a two-factor prod only as num or denom of a fraction
	$= \frac{1/5 \times 3/10}{3/5 \times 7/10 + 1/5 \times 1/3 + 1/5 \times 3/10}$	**M1**	Attempt at P(W2) as sum of appropriate 3 two-factor probs OE seen anywhere
		A1	Unsimplified correct P(W2) num or denom of a fraction
	$= \frac{3/50}{41/75} = 9/82 (0.110)$	**A1**	
	Total:	**4**	

174. 9709_s17_ms_63 Q: 1

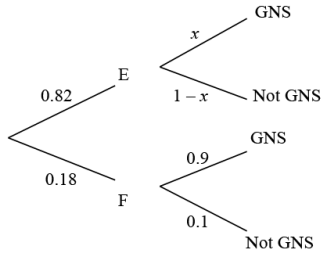
Question	Answer	Marks	Guidance
	P(6) = 0.3	**B1**	SOI
	P(sum is 9) = P(3, 6) + P(4, 5) + P(5, 4) + P(6, 3)	**M1**	Identifying the four ways of summing to 9 (3,6), (6,3) (4,5) and (5,4)
	= (0.03 + 0.02) × 2	**M1**	Mult 2 probs together to find one correct prob of (3,6), (6,3) (4,5) or (5,4) unsimplified
	= 0.1	**A1**	OE
	Total:	**4**	

175. 9709_s17_ms_63 Q: 3

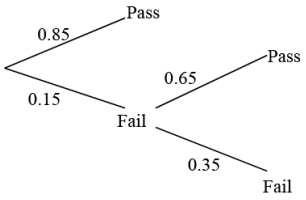
(i)	P(*S*) = 0.65 × 0.6 + 0.35 × 0.75	**M1**	Summing two 2-factor probs or 1 – (sum of two 2-factor probs)
	= 0.653 (261/400)	**A1**	
	Total:	**2**	

Question	Answer	Marks	Guidance
(ii)	$P(Std \mid L) = \frac{P(Std \cap L)}{P(L)} = \frac{0.35 \times 0.25}{1 - 0.6525} = 0.0875/0.3475$	**M1** **M1**	'P(Std)' × 'P(L/Std)'as num of a fraction. Could be from tree diagram in 3(i). Denominator (1 - their (i)) or their (i) **or** 0.65 × 0.4(or 0.6) + 0.35 × 0.25(or 0.75) = 0.26+0.0875 **or** P(L) from their tree diagram
	= 0.252 (35/139)	**A1**	
	Total:	**3**	

176. 9709_w17_ms_61 Q: 5

Question	Answer	Marks	Guidance
(i)		B1	Must see at least 4 probs correct including one with an x in, correct shape
		B1	Shape, clear labels/annotation and all probs correct
		2	
(ii)	$0.82x + 0.18 \times 0.9 = 0.285$	M1	Eqn with x in , two 2-factors on one side
	$x = 0.15$	A1	
		2	
(iii)	$P(E \mid notGNS) = \frac{P(E \cap notGNS)}{P(notGNS)}$	M1	Attempt at P(E∩not GNS) seen as num or denom of fraction
		M1	Attempt at P(not GNS) seen anywhere
	$= \frac{0.82 \times 0.85}{1 - 0.285} = 0.975$	A1	Correct answer
		3	

177. 9709_w17_ms_63 Q: 3

Question	Answer	Marks	Guidance
(i)		M1	Correct shape
		A1	All correct labels and probabilities
		2	

Question	Answer	Marks	Guidance
(ii)	$P(F \mid P) = \frac{P(F \cap P)}{P(P)}$	M1	P(P) consistent with their tree diagram seen anywhere
	$= \frac{0.15 \times 0.65}{0.85 + 0.15 \times 0.65}$ or $\frac{0.15 \times 0.65}{1 - 0.15 \times 0.35}$	A1	Correct unsimplified P(P) seen as num or denom of a fraction
	$= \frac{0.0975}{0.9475}$	M1	P($F \cap P$) found as correct product or consistent with their tree diagram seen as num or denom of a fraction
	$= \frac{39}{379} = 0.103$	A1	
		4	

178. 9709_w17_ms_63 Q: 6

Question	Answer	Marks	Guidance
(i)	${}^{18}P_5$	M1	${}^{18}P_x$ or ${}^{y}P_5$ OE seen, $0 < x < 18$ and $5 < y < 18$, can be mult by $k \geqslant 1$
	= 1 028 160	A1	
		2	

Question	Answer	Marks	Guidance
(ii)	*EITHER:* e.g. ***(CCCCC)********** in 5!×14 ways	**(B1**	5! OE mult by k ⩾ 1, considering the arrangements of cars next to each other
	= 1680	**B1**	Mult by 14 OE, (or 14 on its own) considering positions within the line
	P (next to each other) = 1680/1 028 160	**M1**	Dividing by **(i)** for probability
	P(not next to each other) = 1 – 1680/1 028 160	**M1**	Subtracting prob from 1 (or their ' 5! × 14 ' from **(i)**)
	$= 0.998\left(\frac{611}{612}\right)$ OE	**A1)**	
	OR1: $\frac{5! \times 14!}{18!} = 0.001634$	**(B1**	5! OE mult by k ⩾ 1 (on its own or in numerator of fraction) considering the arrangements of cars next to each other
		B1	Multiply by 14!, (or 14! on its own) considering all ways of arranging spaces with 5 cars together
		M1	Dividing by 18!, total number of ways of arranging spaces
	1 – 0.001634	**M1**	Subtracting prob from 1 (or '5! × 14!' from 18!)
	= 0.998(366)	**A1)**	
	OR2: 4 together – $2\times5!\times14C12 = 21\,840$ 3, 1, 1 – $3\times5!\times14C11 = 131\,040$ 3, 2 – $2\times5!\times14C12 = 21\,840$ 2,2,1 – $3\times5!\times14C11 = 131\,040$ 2,1,1,1 – $4\times5!\times14C10 = 480\,480$ 1,1,1,1,1 – $5!\times14C9 \text{ or } 14P5 = 240\,240$	**(M1**	Listing the six correct scenarios (only): 4 together; 3 together and 2 separate; 3 together and 2 together; two sets of 2 together and 1 separate; 2 together and 3 separate; 5 separate.
		M1	Summing total of the six scenarios, at least 2 correct unsimplified

Question	Answer	Marks	Guidance
	Total = 1 026 480	**A1**	Total of 1 026 480
		M1	Dividing their 1 026 480 by their **6(i)**
	$1\,026\,480 \div 1\,028\,160 = 0.998(366)$	**A1)**	
		5	

Question	Answer	Marks	Guidance
(iii)	R(5) W(4) B(3) Scenarios No. of ways 1 1 1 $= 5\times4\times3 = 60$ 0 1 2 $= 4\times{}^3C_2 = 12$ 0 2 1 $= {}^4C_2\times3 = 18$ 1 0 2 $= 5\times{}^3C_2 = 15$ 2 0 1 $= {}^5C_2\times3 = 30$ 1 2 0 $= 5\times{}^4C_2 = 30$ 2 1 0 $= {}^5C_2\times4 = 40$	**B1**	$5C1\times4C1\times3C1$ or better seen i.e. no. of ways with 3 different colours
		M1	Any of ${}^5C_2$ or ${}^4C_2$ or ${}^3C_2$ seen multiplied by $k > 1$ (can be implied)
		A1	2 correct unsimplified 'no. of ways' other than 5C1 × 4C1 × 3C1
		M1	Summing no more than 7 scenario totals containing at least 6 correct scenarios
	Total = 205	**A1**	
	OR		
	${}^{12}C_3 –$	**M1**	Seeing '${}^{12}C_3 –$', considering all selections of 3 cars
	$– {}^5C_3$	**M1**	Subt ${}^5C_3$ OE, removing only red selections
	$– {}^4C_3$	**M1**	Subt ${}^4C_3$ OE, removing only white selections
	$– {}^3C_3$	**M1**	Subt ${}^3C_3$ OE, removing only black selections
	= 205	**A1**	Correct answer
		5	

179. 9709_m16_ms_62 Q: 3

(i)	P(R) [(1, 4),(2,5), (3,6),(4,7),(5,8)] × 2/64 = 10/64	**M1** **A1**	2	List of at least 4 different options or possibility space diagram Correct answer
(ii)	P(S) = [(3,8)(3,7)(4,8)(4,7)(4,6)(4,5)(5,8) (5,7)(5,6)(6,8)(6,7)(7,8)] × 2 + (5,5)(6,6)(7,7)(8,8) = 28/64	**M1** **A1**	2	List of at least 14 different options or ticks **oe** from possibility space Correct answer
(iii)	P$(R \cap S)$ = 4/64 4/64 ≠ 10/64 × 28/64 Events are not independent	**B1** **M1** **A1**	3	 Comparing their P(R∩S) with (i) ×(ii) with values Correct answer

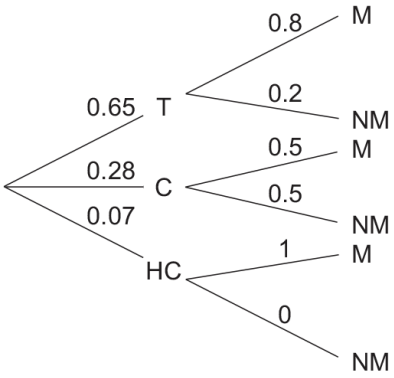
180. 9709_m16_ms_62 Q: 5

(i)	P(Abroad given camping) $= \frac{P(A \cap C)}{P(A \cap C) + P(H \cap C)}$ $= \frac{0.35 \times 0.15}{0.35 \times 0.15 + 0.65 \times 0.4}$ $= \frac{0.0525}{0.3125}$ = 0.168	**M1** **A1** **M1** **A1** **A1**	5	Attempt at P(A∩C) seen alone anywhere Correct answer seen as num or denom of a fraction Attempt at P(C) seen anywhere Correct unsimplified answer seen as num or denom of a fraction Correct answer
(ii)	$(0.65)^n < 0.002$ $n >$ lg (0.002)/lg(0.65) $n = 15$	**M1** **M1** **A1**	3	Eqn with 0.65 or 0.35, power n, 0.002 or 0.998 Attempt to solve their eqn by logs or trial and error need a power Correct answer

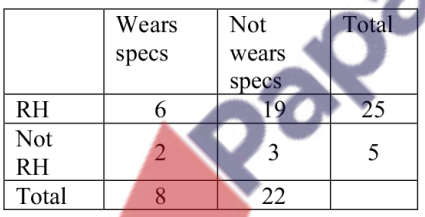
181. 9709_s16_ms_61 Q: 3

(i)	P (cup of coffee) = 0.6×0.9 + 0.4× 0.3 = 0.66	**M1** **A1**	[2]	Summing two 2-factor probabilities Correct answer accept 0.660
(ii)	P(Not on time | no cup of coffee) $= \frac{P(\text{not on time} \cap \text{no cup})}{P(\text{no cup})} = \frac{0.4 \times 0.7}{1 - 0.66}$ $= \frac{0.28}{0.34} = 0.824$	**M1** **M1** **A1**	[3]	0.4×0.7 seen as num or denom of a fraction Attempt at P(no cup) as $0.1 \times p_1 + 0.7 \times p_2$ or as 1 – (i) seen anywhere

182. 9709_s16_ms_62 Q: 1

Qu	Answer	Marks	Notes
(i)		**M1** **A1** [2]	Correct shape with either one branch after HC or 2 branches with 0 prob seen correct Labelled and clear annotation All probs correct
(ii)	$P(C \mid \text{milk}) = \frac{P(\text{coffee} \cap \text{milk})}{P(\text{milk})}$ $= \frac{0.28 \times 0.5}{0.65 \times 0.8 + 0.28 \times 0.5 + 0.07(\times 1)}$ $= \frac{0.14}{0.73}$ $= 0.192$	**M1** **M1** **A1** [3]	Attempt at P(coffee∩ milk)as a two-factor prod only seen as num or denom of a fraction Summing appropriate three 2-factor products seen anywhere (can omit the 1) Correct answer oe

183. 9709_s16_ms_63 Q: 1

Qu	Answer	Marks	Guidance
(i)		**B1** **B1** [2]	One correct row or col including total other than the Total row/column All correct
(ii)	P(X) = 25/30, P(Y) = 8/30 P(X) × P(Y) = 25/30 × 8/30 = 200/900 = 2/9 P($X \cap Y$) = 6/30 = 1/5 ≠ P(X) × P(Y) Not independent	**M1** **M1** **A1** [3]	P(X) or P(Y) from their table or correct from question (denom 30) oe Comparing their P(X) × P(Y) (values substituted) with their evaluated P($X \cap Y$) – not P(X)×P(Y)

184. 9709_w16_ms_61 Q: 6

(i)	P(B, B) = 1/4 ×2/5 = 1/10	**M1** **A1**	[2]	Multiplying two different probs
(ii)	$P(X = 1) = P(R,R) + P(B,B)$ = 3/4 × 4/5 + 1/10 = 14/20 (7/10)	**M1** **M1** **A1**	[3]	Finding P(R, R) (=3/5) Summing two options
(iii)	P(B B) $= \frac{P(B \cap B)}{P(B)} = \frac{1/10}{3/4 \times 1/5 + 1/4 \times 2/5}$ = 2/5	**M1** **M1** **A1** **A1**	[4]	their (i) seen as num or denom of a fraction ¾ × p_1 + ¼ × p_2 seen anywhere 1/4 (unsimplified) seen as num or denom of a fraction, www

185. 9709_w16_ms_62 Q: 1

	P(C given L) = $\frac{P(C \cap L)}{P(L)}$ $= \frac{0.65 \times 0.1}{0.65 \times 0.1 + 0.3 \times 0.15 + 0.05 \times 0.6}$ $= \frac{0.065}{0.14}$ = 0.464, $\frac{13}{28}$	**M1** **A1** **M1** **A1** **A1**	[5]	P($C \cap L$) seen as num or denom of a fraction Correct unsimplified P($C \cap L$) as numerator Summing three 2-factor products seen anywhere 0.14 (unsimplified) seen as num or denom of a fraction oe

186. 9709_w16_ms_63 Q: 4

(i)	64/250, 0.256	**B1**	[1]	oe
(ii)	190/250, 0.76(0)	**B1**	[1]	oe

(iii)	P(X) = 80/250 = 8/25 P(Y) = 100/250 = 2/5 P ($X \cap Y$) = 32/250 = 16/125 P(X) × P(Y) = $\frac{8}{25} \times \frac{2}{5} = \frac{16}{125}$ Since P(X) × P(Y) = P ($X \cap Y$) therefore independent	**M1** **M1** **B1** **M1** **A1**	[5]	attempt at P(X) attempt at P(Y) oe comparing P(X) × P(Y) and P($X \cap Y$) so long as independence has not been assumed correct answer with all working correct

187. 9709_s15_ms_61 Q: 3

(i)	$P(A) = \frac{1}{3} \times \frac{2}{3} + \frac{2}{3} \times \frac{1}{3} = \frac{4}{9}$ $P(B) = \frac{27}{36} = \frac{3}{4}$ $P(A \cap B) = \frac{12}{36} = \frac{1}{3}$ $P(A) \times P(B) = \frac{4}{9} \times \frac{3}{4} = \frac{1}{3}$ Independent as $P(A \cap B) = P(A) \times P(B)$	**M1** **M1** **B1** **M1** **A1** **[5]**	Sensible attempt at P(A) Sensible attempt at P(B) correct P($A \cap B$) Cf P($A \cap B$) with P(A)×P(B) need at least 1 correct Correct conclusion following all correct working
(ii)	Not mutually exclusive because $P(A \cap B) \neq 0$ Or give counter example e.g. 1 and 6	**B1**√ **[1]**	ft their P($A \cap B$)

188. 9709_s15_ms_61 Q: 4

(i)	$(1 - x)0.9 + x \times 0.24 = 0.801$ $x = 0.15$	**M1** **A1** **A1** **[3]**	Eqn with sum of two 2-factor probs = 0.801 Correct equation Correct answer

(ii)	P(⩾100 times given ⩽ 3 views) $= \frac{P(\geqslant 100 \text{ times} \cap \geqslant 3 \text{ views})}{P(\geqslant 3 \text{ views})} =$ $\frac{0.85 \times 0.1}{0.85 \times 0.1 + 0.15 \times 0.76 \text{ or } 1 - 0.801}$ $= 0.427$	**B1** **M1** **A1** **A1** **[4]**	0.85×0.1 seen on its own as num or denom of a fraction Attempt at P(⩾ 3 views) either $(0.85 \times p_1 + 0.15 \times p_2)$ or $1 - 0.801$ seen anywhere Correct unsimplified P(⩾ 3 views) as num or denom of a fraction Correct answer

189. 9709_s15_ms_62 Q: 4

(i)	$0.3 \times 0.72 + 0.7 \times x = 0.783$ $x = 0.81$	M1 A1 A1	**3**	Eqn with sum of two 2-factor probs =0.783 Correct equation Correct answer

(ii)	P(S given not like) $= \frac{P(S \cap NL)}{P(NL)}$ $= \frac{0.3 \times 0.28}{0.3 \times 0.28 + 0.7 \times 0.19 \text{ or } 1 - 0.783}$ $= 0.387$ (12/31)	B1 M1 A1 A1	**4**	0.3×0.28 seen on its own as num or denom of a fraction Attempt at P(NL) either $(0.3 \times p_1) + (0.7 \times p_2)$ or $1 - 0.783$ seen anywhere Correct unsimplified P(NL) as num or denom of a fraction Correct answer

190. 9709_w15_ms_62 Q: 2

	P(no men) $\frac{{}^9C_6}{{}^{16}C_6} = \frac{84}{8008} = \frac{21}{2002} = \frac{3}{286}$ $= 0.0105$ OR $\frac{9}{16} \times \frac{8}{15} \times \frac{7}{14} \times \frac{6}{13} \times \frac{5}{12} \times \frac{4}{11} = 0.0105$	**B1** **B1** **B1** 3 **B1** **B1** **B1**	${}^9C_6$ seen anywhere ${}^{16}C_6$ seen as denom of fraction oe Correct final answer $(9 \times 8 \times 7 \times 6 \times 5 \times 4)$ seen anywhere Correct unsimplified denom Correct final answer

191. 9709_w15_ms_63 Q: 2

(i)	$P(X) = \frac{20}{28}\left(\frac{5}{7}\right)(0.714), 71.4\%$	**B1** 1		oe
(ii)	$P(F) = \frac{20}{28} \times \frac{1}{4} \times \frac{8}{28} \times \frac{6}{10} = \frac{7}{20}$	**M1** **A1** 2		Summing two 2-factor probs created by One of ¼ or ¾ multiplied by 20/28 or 8/28 Added to 4/10 or 6/10 × altn population prob Correct answer
(iii)	$P(X \mid F) = \frac{5/28}{7/20} = \frac{25}{49}(0.510)$	**M1** **A1** 2		Their unsimplified country X probability (5/28) as num or denom of a fraction Or (their fair hair population) ÷ (total fair hair pop) Correct answer

192. 9709_w15_ms_63 Q: 3

(i)	$P(S) = \frac{3}{16}$ $P(T) = \frac{4}{16}$ $P(S \cap T) = \frac{2}{16}$ $P(S) \times P(T) = \frac{3}{64} \neq \frac{2}{16}$ Not independent	**M1** **M1** **B1** **M1** **A1** 5		Sensible attempt at P(S) Sensible attempt at P(T) Correct P($S \cap T$) comp P(S) × P(T) with P($S \cap T$) (their values), evaluated Correct conclusion following all correct working
(ii)	not exclusive since $P(S \cap T) \neq 0$ Or counter example e.g. 1 and 3 Or P(SUT) ≠ P(S)+P(T) with values	**B1**√ 1		FT their P($S \cap T$), not obtained from P(S) × P(T), with value and statement.

193. 9709_m22_ms_52 Q: 1

Question	Answer	Marks	Guidance
(a)	X | −2 | −1 | 0 | 1 | 2 P(X) | $\frac{1}{16}$ | $\frac{3}{16}$ | $\frac{5}{16}$ | $\frac{5}{16}$ | $\frac{2}{16}$ 0.0625 | 0.1875 | 0.3125 | 0.3125 | 0.125	**B1** **B1** **B1**	Table with correct X values and at least one probability $0 < p < 1$. Condone any additional X values if probability stated as 0. No repeated X values. 3 correct probabilities linked with correct outcomes, may not be in table. 2 further correct probabilities linked with correct outcomes, may not be in table No repeated X values. **SC** if less than 3 correct probabilities seen, award **SCB1** Sum of *their* probabilities, $0 < p < 1$, of 4,5 or 6 X values = 1 (condone summing to 1±0·01 or better).
		3	
(b)	$\left[\frac{1}{16}\times -2^2+\frac{3}{16}\times -1^2+\left(\frac{5}{16}\times 0^2\right)+\frac{5}{16}\times 1^2+\frac{2}{16}\times 2^2-\left(\frac{1}{4}\right)^2\right]$ $\frac{1\times 4+3\times 1+5\times 0+5\times 1+2\times 4}{16}-0.25^2$	**M1**	Appropriate variance formula using $(E(X))^2$ value, accept unsimplified. FT *their* table with at least 3 different X values even if probabilities not summing to 1, $0 < p < 1$. Condone 1 error providing all probabilities <1 and 0.25^2 used
	$\left[=\frac{5}{4}-\frac{1}{16}=\right]\frac{19}{16}$, 1.1875	**A1**	Condone 1.188 or 1.19 WWW
		2	

194. 9709_m22_ms_52 Q: 2

Question	Answer	Marks	Guidance
(a)	[P(>2) = 1 – P(0,1,2) =] $1-({}^7C_0\ 0.18^0\ 0.82^7+{}^7C_1\ 0.18^1\ 0.82^6+{}^7C_2\ 0.18^2\ 0.82^5)$	**M1**	One term ${}^7C_x\ p^x(1-p)^{7-x}$, $0<p<1, 0<x<7$
	= 1 – (0.249285 + 0.383048 + 0.252251) = 1 – 0.88458	**A1**	Correct unsimplified expression or better Condone omission of brackets if recovered
	0.115	**B1**	WWW. 0·115 ⩽ p < 0·1155 not from wrong working
		3	
(b)	[P(at least 1 day of rain) = 1 – P(0) = $1-(0.82)^7$ =] 0.7507	**B1**	AWRT 0.751 seen
	[P(exactly 2 periods) =] $0.7507^2\times(1-0.7507)\times 3$	**M1**	FT *their* $1-p^7$ or *their* 0.7507 if identified, not 0.18, 0.82 Accept $\times{}^3C_r$, r=1,2 or $\times{}^3P_1$ for ×3 Condone ×2
	0.421	**A1**	Accept 0.421 ⩽ p ⩽ 0.4215 **SC B1** if 0/3 scored for final answer only 0.421 ⩽ p ⩽ 0.4215
		3	

195. 9709_m22_ms_52 Q: 6

Question	Answer	Marks	Guidance
(a)	[Probability of lemon = $\frac{3}{15}=\frac{1}{5}$] $\left[\left(\frac{4}{5}\right)^6\times\frac{1}{5}=\right]\frac{4096}{78125}, 0.0524$	**B1**	0.0524288 rounded to more than 3SF if final answer
		1	
(b)	$\left(1-\frac{1}{5}\right)^6$	**M1**	or $\left(\frac{4}{5}\right)^6$. FT *their* $\frac{1}{5}$ or correct. From final answer Condone $\left(\frac{4}{5}\right)^5$ or $\left(\frac{1}{5}\right)\times\left(\frac{4}{5}\right)^5+\left(\frac{4}{5}\right)^6$
	$\frac{4096}{15625}$, 0.262	**A1**	0.262144 rounded to more than 3SF
	Alternative method for question 6(b)		
	[1 – P(1,2,3,4,5,[6]) =] $1-\left(\frac{1}{5}+\frac{4}{5}\times\frac{1}{5}+\left(\frac{4}{5}\right)^2\times\frac{1}{5}+\left(\frac{4}{5}\right)^3\times\frac{1}{5}+\left(\frac{4}{5}\right)^4\times\frac{1}{5}+\left(\frac{4}{5}\right)^5\times\frac{1}{5}\right)$	**M1**	From final answer Condone omission of $\left(\frac{4}{5}\right)^5\times\frac{1}{5}$
	$\frac{4096}{15625}$, 0.262	**A1**	0.262144 rounded to more than 3SF
		2	

Question	Answer	Marks	Guidance
(c)	$\frac{10}{15}\times\frac{9}{14}\times\frac{8}{13}$	**M1**	$\frac{a}{15}\times\frac{a-1}{14}\times\frac{a-2}{13}$, no additional terms
	$\frac{24}{91}$, 0·264	**A1**	0.263736 rounded to more than 3SF
	Alternative method for question 6(c)		
	$\frac{3}{15}\times\frac{2}{14}\times\frac{1}{13}+3\times\frac{3}{15}\times\frac{2}{14}\times\frac{7}{13}+3\times\frac{3}{15}\times\frac{7}{14}\times\frac{6}{13}+\frac{7}{15}\times\frac{6}{14}\times\frac{5}{13}$	**M1**	[3Ls + 2Ls1S + 1L2Ss + 3Ss] Condone one numerator error. Condone no multiplications seen if tree diagram complete with probabilities on each branch, scenarios listed and attempt at evaluation
	$\frac{24}{91}$, 0·264	**A1**	0.263736 rounded to more than 3SF
	Alternative method for question 6(c)		
	$1-\left(\frac{5}{15}\times\frac{4}{14}\times\frac{3}{13}+3\times\frac{5}{15}\times\frac{4}{14}\times\frac{10}{13}+3\times\frac{5}{15}\times\frac{10}{14}\times\frac{9}{13}\right)$	**M1**	1 – P(3,2,1 oranges) Condone one numerator error.
	$\frac{24}{91}$, 0·264	**A1**	0.263736 rounded to more than 3SF
	Alternative method for question 6(c)		
	$\frac{{}^{10}C_3}{{}^{15}c_3}$	**M1**	
	$\frac{24}{91}$, 0·264	**A1**	0.263736 rounded to more than 3SF
		2	

Question	Answer	Marks	Guidance
(d)	$\frac{7}{15}\times\frac{5}{14}\times\frac{3}{13}\times 3!$	M1	All probabilities of the form: $\frac{7}{a}\times\frac{5}{b}\times\frac{3}{c}$, $13 \leqslant a,b,c \leqslant 15$
		M1	$\frac{e}{f}\times\frac{g}{h}\times\frac{i}{j}\times 3!$ e,f,g,h,i,j positive integers forming probabilities or 6 identical probability calculations or values added, no additional terms
	$\frac{3}{13}$, 0·231	A1	0·230769 rounded (not truncated) to more than 3SF
	Alternative method for question 6(d)		
	$\frac{{}^3C_1\times{}^5C_1\times{}^7C_1}{{}^{15}C_3}$	M1	$\frac{{}^3C_1\times{}^5C_1\times{}^7C_1}{k}$, k integer > 1 Condone use of permutations
		M1	$\frac{{}^3C_a\times{}^5C_b\times{}^7C_c}{{}^{15}C_3}$, $0<a<3$, $0<b<5$, $0<c<7$, Condone use of permutations
	$\frac{3}{13}$, 0·231	A1	0·230769 rounded (not truncated) to more than 3SF
		3	

Question	Answer	Marks	Guidance
(e)	$\frac{\frac{7}{15}\times\frac{6}{14}\times\frac{5}{13}+\frac{3}{15}\times\frac{7}{14}\times\frac{6}{13}\times 3}{their(c)}$ $\left[=\frac{14}{65}\div\frac{24}{91}\right]$	B1	$\frac{3}{15}\times\frac{7}{14}\times\frac{6}{13}\times 3$ seen (SSL, SLS, LSS) **SC B1** $\frac{3}{65}\times 3, \frac{126}{2730}\times 3$ seen
		B1	$\frac{7}{15}\times\frac{6}{14}\times\frac{5}{13}$ seen in numerator (SSS) **SCB1** $\frac{210}{2730}, \frac{1}{13}$ seen in numerator
		M1	Fraction with *their* **(c)** or correct in denominator $\left(\frac{720}{2730}, \frac{24}{91}, 0.263736\right)$
	$=\frac{49}{60}$, 0·817	A1	Accept 0.816
	Alternative method for question 6(e)		
	$\frac{{}^7C_2\times{}^3C_1+{}^7C_3}{{}^{10}C_3}$	B1	${}^7C_2\times{}^3C_1$ seen (SSL, SLS, LSS) **SCB1** 21 × 3 seen or use of permutations
		B1	${}^7C_3$ seen in numerator (SSS) **SCB1** 35 seen in numerator or use of permutations
		M1	Fraction with ${}^{10}C_3$ or consistent with *their* numerator of **6(c)** in denominator
	$=\frac{49}{60}$, 0·817	A1	Accept 0.816
		4	

196. 9709_s22_ms_51 Q: 4

Question	Answer	Marks	Guidance
(a)	$a = \text{P(1 head)} = 0.7\times(0.5)^3 + 0.3\times(0.5)^3\times 3 = \frac{1}{5}$	B1	Clear statement of unevaluated correct calculation $=\frac{1}{5}$. AG
	$b = 0.7\times 0.5^3\times 3 + 0.3\times 0.5^3\times 3 = \frac{3}{8}$	M1	Clear statement of unevaluated calculation for either b or c
	$c = 0.7\times 0.5^3\times 3 + 0.3\times 0.5^3 = \frac{3}{10}$	A1	For either b or c correct
	$\left[or\ c = \frac{27}{40} - b\right]$	B1 FT	*their* b + *their* $c = \frac{27}{40}$
		4	
(b)	$\left[\text{E}(X) = \frac{3\times 0+16\times 1+30\times 2+24\times 3+7\times 4}{80} = \right]\frac{176}{80}$ or 2.2	B1 FT	Correct or accept unsimplified calculation using *their* values for b and c seen (sum of probabilities = 1)
		1	

Question	Answer	Marks	Guidance
(c)	$[\text{P}(0, 1, 2) =]\ {}^{10}C_0\ 0.2^0\ 0.8^{10} + {}^{10}C_1\ 0.2^1\ 0.8^9 + {}^{10}C_2\ 0.2^2\ 0.8^8$	M1	One term ${}^{10}C_x\ p^x(1-p)^{10-x}$, for $0 < x < 10,\ 0 < p < 1$
	0.107374 + 0.268435 + 0.301989	A1	Correct expression, accept unsimplified leading to final answer
	0.678	B1	$0.677 < p \leqslant 0.678$
	Alternative method for question 4(c)		
	$1 - [{}^{10}C_{10}\ 0{\cdot}2^{10}0.8^0 + {}^{10}C_9\ 0{\cdot}2^9 0.8^1 + {}^{10}C_8\ 0{\cdot}2^8 0.8^2 + {}^{10}C_7\ 0{\cdot}2^7 0.8^3 + {}^{10}C_6\ 0{\cdot}2^6 0.8^4 + {}^{10}C_5\ 0{\cdot}2^5 0.8^5 + {}^{10}C_4\ 0{\cdot}2^4 0.8^6 + {}^{10}C_3\ 0{\cdot}2^3 0.8^7]$	M1	One term ${}^{10}C_x\ p^x(1-p)^{10-x}$, for $0 < x < 10,\ 0 < p < 1$
		A1	Correct expression, accept unsimplified
	0.678	B1	$0.677 < p \leq 0.678$
		4	
(d)	$0.8^6\times 0.2 + 0.8^7\times 0.2 = 0.0524288 + 0.041943$	M1	$p^l\times(1-p) + p^m\times(1-p),\ l = 6, 7$ $m = l + 1,\ 0 < p < 1$
	0.0944	A1	$0.09437 \leqslant p \leqslant 0.0944$
		2	

197. 9709_s22_ms_52 Q: 2

Question	Answer	Marks	Guidance
(a)	x: 2, 3, 4, 5, 6 p: $\frac{1}{36}$, $\frac{4}{36}$, $\frac{10}{36}$, $\frac{12}{36}$, $\frac{9}{36}$ 0.02778, 0.1111, 0.2778, 0.3333, 0.25	B1	Table with correct X values and at least one probability. Condone any additional X values if probability stated as 0.
		B1	3 correct probabilities linked with correct outcomes. Accept 3 sf decimals.
		B1	2 further correct probabilities linked with correct outcomes. Accept 3 sf decimals.
		3	**SC B1** for 5 probabilities ($0 < p < 1$) that sum to 1 with less than 3 correct probabilities.

Question	Answer	Marks	Guidance
(b)	If method FT from *their* incorrect **(a)**, expressions for E(X) and Var(X) must be seen at the stage shown in **bold** (or less simplified) in the scheme with all probabilities < 1.		
	$\left[\mathrm{E}(X)=\frac{1\times2+4\times3+10\times4+12\times5+9\times6}{36}=\right]$ $\frac{\mathbf{2+12+40+60+54}}{\mathbf{36}}$	**M1**	Accept unsimplified expression. May be calculated in variance. FT *their* table with 4 or more probabilities summing to 0.999 ⩽ total ⩽ 1 ($0 < p < 1$).
	$\left[\mathrm{Var}(X)=\frac{1\times2^2+4\times3^2+10\times4^2+12\times5^2+9\times6^2}{36}-(\textit{their}\,\mathrm{E}(X))^2=\right]$ $\frac{\mathbf{1\times4+4\times9+10\times16+12\times25+9\times36}}{\mathbf{36}}-\left(\textit{their}\,\frac{\mathbf{14}}{\mathbf{3}}\right)^2$ $\left[\frac{4+36+160+300+324}{36}-\left(\textit{their}\,\frac{14}{3}\right)^2\right]$	**M1**	Appropriate variance formula using *their* $(\mathrm{E}(X))^2$ value. FT *their* table with 3 or more probabilities ($0 < p < 1$) which need not sum to 1 and the calculation in **bold** (or less simplified) seen.
	$\mathrm{E}(X)=\frac{168}{36}$, $\frac{14}{3}$, 4.67 $\mathrm{Var}(X)=\frac{10}{9}$, $1\frac{1}{9}$, 1.11, $\frac{1440}{1296}$	**A1**	Answers for E(X) and Var(X) must be identified. E(X) may be identified by correct use in Variance. Condone E, V, μ, σ^2 etc. If M0 earned **SC B1** for identified correct final answers.
		3	

198. 9709_s22_ms_53 Q: 3

Question	Answer	Marks	Guidance
(a)	$k=\frac{1}{18}$ ($4k+k+4k+9k=18k=1$)	**B1**	SOI
	x: −2, 1, 2, 3 P($X=x$): $\frac{4}{18}$, $\frac{1}{18}$, $\frac{4}{18}$, $\frac{9}{18}$	**M1**	Table with correct x values and at least one probability accurate using *their* k. Values need not be in order, lines may not be drawn, may be vertical, x and P($X=x$) may be omitted. Condone any additional X values if probability stated as 0.
		A1	Remaining probabilities correct.
		3	

Question	Answer	Marks	Guidance
(b)	$\left[\mathrm{E}(X)=\frac{4\times-2+1\times1+4\times2+9\times3}{18}=\right]$ $\frac{-8+1+8+27}{18}$	**M1**	$-8k+k+8k+27k$ May be implied by use in Variance. Accept unsimplified expression. FT *their* table if probabilities sum to 1 or 0.999. **SC B1** 28k.
	$\left[\mathrm{Var}(X)=\frac{4\times(-2)^2+1\times1^2+4\times2^2+9\times3^2}{18}-(\textit{their}\,\mathrm{E}(X))^2=\right]$ $=\frac{16+1+16+81}{18}-\left(\textit{their}\,\frac{28}{18}\right)^2$	**M1**	$16k+k+16k+81k-(\textit{their}\ \text{mean})^2$ FT *their* table even if probabilities not summing to 1. Note: If table is correct, $\frac{114}{18}-(\textit{their}\,\mathrm{E}(X))^2$ M1. **SC B1** $114k-(\textit{their}\ \text{mean})^2$.
	E(X) = $\frac{14}{9}$, $1\frac{5}{9}$, 1.56, Var (X) = $\frac{317}{81}$, $3\frac{74}{81}$, 3.91	**A1**	Answers for E(X) and Var(X) must be identified. 3.91 ⩽ Var(X) ⩽ 3.914
		3	

199. 9709_s22_ms_53 Q: 4

Question	Answer	Marks	Guidance
(a)	$\left[\left(\frac{5}{6}\right)^7\times\frac{1}{6}=\right]0.0465, \frac{78125}{1679616}$	**B1**	$0.0465 \leqslant p < 0.04652$
		1	

Question	Answer	Marks	Guidance
(b)	$P(X<6)=1-\left(\frac{5}{6}\right)^5$ or $\frac{1}{6}+\left(\frac{5}{6}\right)\left(\frac{1}{6}\right)+\left(\frac{5}{6}\right)^2\left(\frac{1}{6}\right)+\left(\frac{5}{6}\right)^3\left(\frac{1}{6}\right)+\left(\frac{5}{6}\right)^4\left(\frac{1}{6}\right)$	**M1**	$1-p^n$, $0<p<1$, $n=4, 5, 6$ or sum of 4, 5 or 6 terms $p\times(1-p)^n$ for $n=0,1,2,3,4(5)$.
	0.598, $\frac{4651}{7776}$	**A1**	
		2	
(c)	[Probability of total less than 4 is] $\frac{3}{36}$ or $\frac{1}{12}$	**B1**	SOI
	[1 – P(0, 1, 2)] $=1-\left({}^{10}C_0\left(\frac{1}{12}\right)^0\left(\frac{11}{12}\right)^{10}+{}^{10}C_1\left(\frac{1}{12}\right)^1\left(\frac{11}{12}\right)^9+{}^{10}C_2\left(\frac{1}{12}\right)^2\left(\frac{11}{12}\right)^8\right)$	**M1**	One term ${}^{10}C_x\ p^x(1-p)^{10-x}$, for $0<x<10$, $0<p<1$.
	1 – (0.418904 + 0.380822 + 0.155791)	**A1 FT**	Correct expression. Accept unsimplified.
	0.0445	**A1**	$0.04448 \leqslant p \leqslant 0.0445$
		4	

200. 9709_m21_ms_52 Q: 1

Question	Answer	Marks	Guidance
(a)	$\left[\left(\frac{4}{5}\right)^7\frac{1}{5}=\right]\ \frac{16384}{390625}$ or 0·0419[43…]	**B1**	Evaluated, final answer.
		1	
(b)	$1-\left(\frac{4}{5}\right)^5$ or $\frac{1}{5}+\frac{4}{5}\times\frac{1}{5}\left(\frac{4}{5}\right)^2\times\frac{1}{5}+\left(\frac{4}{5}\right)^3\times\frac{1}{5}+\left(\frac{4}{5}\right)^4\times\frac{1}{5}$	**M1**	$1-p^n$ $n=5,6$ or $p+pq+pq^2+pq^3+pq^4\ (+pq^5)$ $0<p<1$, $p+q=1$, Sum of a geometric series may be used.
	$\frac{2101}{3125}$ or 0·672[32]	**A1**	Final answer.
	Alternative method for question 1(b)		
	[P(at least 1 three scored in 5 throws) =] $\left(\frac{1}{5}\right)^5+{}^5C_4\left(\frac{1}{5}\right)^4\left(\frac{4}{5}\right)+{}^5C_3\left(\frac{1}{5}\right)^3\left(\frac{4}{5}\right)^2+{}^5C_2\left(\frac{1}{5}\right)^2\left(\frac{4}{5}\right)^3+{}^5C_4\left(\frac{1}{5}\right)\left(\frac{4}{5}\right)^4$	**M1**	$(p)^5+{}^5C_4(p)^4(q)+{}^5C_3(p)^3(q)^2+{}^5C_2(p)^2(q)^3+{}^5C_1(p)(q)^4$ or $(p)^6+{}^6C_5(p)^5(q)+{}^6C_4(p)^4(q)^2+{}^6C_3(p)^3(q)^3$ $+{}^6C_2(p)^2(q)^4+{}^6C_1(p)(q)^5$, $0<p<1, p+q=1$ At least first, last and one intermediate term is required to show pattern of terms if not all terms stated.
	$\frac{2101}{3125}$ or 0·672[32]	**A1**	Final answer.
		2	

201. 9709_m21_ms_52 Q: 4

Question	Answer	Marks	Guidance
(a)	x | 1 | 2 | 3 | 4 prob | $4k$ | $6k$ | $6k$ | $4k$	**B1**	Table with × values and one correct probability expressed in terms of k. Condone any additional × values if probability stated as 0.
		B1	Remaining 3 probabilities correct expressed in terms of k – condone if the first correct probability is not in table.
		2	
(b)	$[4k+6k+6k+4k=1]\ k=\frac{1}{20}\quad(=0.05)$	**B1**	Correct value for k SOI. May be calculated in **4(a)**. **SC B1** If denominator $20k$ used throughout.
	$E(X)=1\times\frac{4}{20}+2\times\frac{6}{20}+3\times\frac{6}{20}+4\times\frac{4}{20}=\frac{4}{20}+\frac{12}{20}+\frac{18}{20}+\frac{16}{20}$ $(=2.5)$	**M1**	Accept unsimplified expression. Condone $4k + 12k + 18k + 16k$. May be implied by use in Variance expression. **Special ruling:** Allow use of denominator $20k$.
	$\text{Var}(X)=1^2\times\frac{4}{20}+2^2\times\frac{6}{20}+3^2\times\frac{6}{20}+4^2\times\frac{4}{20}-\left(\textit{their}\ 2\frac{1}{2}\right)^2$ $=(4+24+54+64)\times\textit{their}\,0.05-(\textit{their}\,2.5)^2$ **Or** $(1-2.5)^2\times\frac{4}{20}+(2-2.5)^2\times\frac{6}{20}+(3-2.5)^2\times\frac{6}{20}+(4-2.5)^2\times\frac{4}{20}$	**M1**	Appropriate variance formula with *their* numerical probabilities using *their* $(\text{E}(X))^2$, accept unsimplified, with *their* k substituted. **Special ruling:** If denominator $20k$ used throughout, accept appropriate variance formula in terms of k.
	1·05	**A1**	AG, NFWW.
		4	

202. 9709_s21_ms_51 Q: 7

Question	Answer	Marks	Guidance
(a)	$P(X=3)=\frac{4}{7}\times\frac{3}{6}\times\frac{3}{5}$	M1	$\frac{m}{7}\times\frac{n}{6}\times\frac{o}{5}$ used throughout. condone use of $\frac{1}{2}$
	$\frac{6}{35}$	A1	AG. The fractions must be identified, e.g. P(NC, NC, C), may be seen in a tree diagram.
		2	

Question	Answer	Marks	Guidance
(b)	x: 1, 2, 3, 4, 5 p: $\frac{15}{35}$, $\frac{10}{35}$, $\frac{6}{35}$, $\frac{3}{35}$, $\frac{1}{35}$	B1	Table with x values and at least one probability Condone any additional x values if probability stated as 0.
		B1	One correct probability other than $X = 3$ linked to the correct outcome
		B1	Two further correct probabilities other than $X = 3$ seen linked to the correct outcome
		B1FT	All probabilities correct, or at least 4 probabilities summing to 1
		4	
(c)	$[E(X) = 1\times\frac{15}{35} + 2\times\frac{10}{35} + 3\times\frac{6}{35} + 4\times\frac{3}{35} + 5\times\frac{1}{35}]$ $E(X) = \frac{15+20+18+12+5}{35}\left[= \frac{70}{35} = 2\right]$	M1	At least 4 correct terms FT *their* values in **(a)** with probabilities summing to 1 May be implied by use in Variance, accept unsimplified expression.
	$\text{Var}(X) = \left[\frac{1^2\times15 + 2^2\times10 + 3^2\times6 + 4^2\times3 + 5^2\times1}{35} - 2^2 =\right]$ $\frac{15+40+54+48+25}{35} - 2^2$	M1	Appropriate variance formula using *their* $(E(X))^2$. FT *their* table accept probabilities not summing to 1.
	$\left[= \frac{182}{35} - 4\right] = \frac{6}{5}$	A1	**N.B.** If method FT for M marks from *their* incorrect **(b)**, expressions for E(X) and Var(X) must be seen unsimplified with all probabilities <1
		3	

203. 9709_s21_ms_52 Q: 1

Question	Answer	Marks	Guidance
(a)	6	B1	WWW
		1	
(b)	$\left(\frac{5}{6}\right)^3\frac{1}{6} + \left(\frac{5}{6}\right)^4\frac{1}{6} + \left(\frac{5}{6}\right)^5\frac{1}{6} + \left(\frac{5}{6}\right)^6\frac{1}{6}$	M1	$p^3(1-p) + p^4(1-p) + p^5(1-p) + p^6(1-p)$, $0 < p < 1$
	0.300 (0.2996…)	A1	At least 3s.f. Award at most accurate value.
	Alternative method for Question 1(b)		
	$\left(\frac{5}{6}\right)^3 - \left(\frac{5}{6}\right)^7$	M1	$p^3 - p^7$, $0 < p < 1$
	0.300 (0.2996…)	A1	At least 3s.f. Award at most accurate value.
		2	
(c)	$1 - \left(\frac{5}{6}\right)^9$	M1	$1 - p^n$, $0 < p < 1$, $n = 9, 10$
	0.806	A1	
	Alternative method for Question 1(c)		
	$\frac{1}{6} + \frac{1}{6}\left(\frac{5}{6}\right) + \frac{1}{6}\left(\frac{5}{6}\right)^2 + \cdots + \frac{1}{6}\left(\frac{5}{6}\right)^8$	M1	$p + p(1-p) + p(1-p)^2 + p(1-p)^3 + p(1-p)^4 + p(1-p)^5 + p(1-p)^6 + p(1-p)^7 + p(1-p)^8 (+ p(1-p)^9)$, $0 < p < 1$ As per answer for minimum terms shown
	0.806	A1	
		2	

204. 9709_s21_ms_52 Q: 4

Question	Answer	Marks	Guidance
(a)	X: –1, 0, 1, 2, 3 $P(X)$: $\frac{1}{9}$, $\frac{2}{9}$, $\frac{1}{9}$, $\frac{3}{9}$, $\frac{2}{9}$	B1	Table with correct X values and at least one probability Condone any additional X values if probability stated as 0.
		B1	2 correct probabilities linked with correct outcomes, may not be in table.
		B1	3 further correct probabilities linked with correct outcomes, may not be in table. **SC** if less than 2 correct probabilities seen, award **SCB1** for sum of *their* 4 or 5 probabilities in table = 1
		3	
(b)	$\left[E(X)=\frac{-1\times1+(0\times2)+1\times1+2\times3+3\times2}{9}=\right]$ $\frac{-1+1+6+6}{9}$	M1	May be implied by use in variance, accept unsimplified expression. FT *their* table if *their* 3 or more probabilities sum to 1 or 0.999
	[Var (X) =] $\left[\frac{-1^2\times1+(0^2\times2)+1^2\times1+2^2\times3+3^2\times2}{9}-(\textit{their}\ E(X))^2\right]$ $\frac{1+0+1+12+18}{9}-(\textit{their}\ E(X))^2$	M1	Appropriate variance formula using *their* $(E(X))^2$ value. FT *their* table even if *their* 3 or more probabilities not summing to 1.
	$E(X)=\frac{4}{3}$ or 1.33 and $Var(X)=\frac{16}{9}$ or 1.78	A1	Answers for E(X) and Var(X) must be identified
		3	**N.B.** If method FT for M marks from *their* incorrect **(b)**, expressions for E(X) and Var(X) must be seen unsimplified with all probabilities <1

205. 9709_s21_ms_53 Q: 2

Question	Answer	Marks	Guidance
2	$p + p + 0.1 + q + q = 1$	**B1**	Sum of probabilities = 1
	$0.1 + 2q = 3(2p)$	**B1**	Use given information
	Attempt to solve two correct equations in p and q	**M1**	**Either** use of Substitution method to form a single equation in either p or q and finding values for both unknowns. **Or** use of Elimination method by writing both equations in same form (usually $ap+bq=c$) and + or – to find an equation in one unknown and finding values for both unknowns.
	$p=\frac{1}{8}$ or 0.125 and $q=\frac{13}{40}$ or 0.325	**A1**	CAO, both WWW
		4	

206. 9709_s21_ms_53 Q: 4

Question	Answer	Marks	Guidance
(a)	[Possible cases: 1 1 2, 1 2 1, 2 1 1] Probability $= \left(\frac{1}{6}\right)^3 \times 3$	M1	$\left(\frac{1}{6}\right)^3 \times k$, where k is an integer.
		M1	Multiply a probability by 3, not +, – or ÷
	$\frac{1}{72}$	A1	Accept $\frac{3}{216}$ or $0.013\dot{8}$ or 0.0139
		3	
(b)	$P(18) = \left(\frac{1}{6}\right)^3 \left[= \frac{1}{216}\right]$	B1	
	P(18 on 5th throw) $= \left(\frac{215}{216}\right)^4 \times \frac{1}{216}$	M1	$(1-p)^4 p$, $0 < \text{their } p < 1$
	0.00454	A1	
		3	

207. 9709_w21_ms_51 Q: 1

Question	Answer	Marks	Guidance
(a)	$\left(\frac{3}{4}\right)^6 \frac{1}{4}$	M1	$(1-p)^6 p$, $0 < p < 1$
	0.0445, $\frac{729}{16384}$	A1	
		2	
(b)	$\left(\frac{3}{4}\right)^9$	M1	$\left(\frac{3}{4}\right)^n$ *or* p^n, $0 < p < 1, n = 8,9,10$
	0.0751, $\frac{19683}{262144}$	A1	
		2	

208. 9709_w21_ms_51 Q: 4

Question	Answer	Marks	Guidance
(a)	x: -1 | 0 | 1 | 2 | 3 p: $\frac{1}{12}=0.0833$ | $\frac{2}{12}=0.167$ | $\frac{4}{12}=0.333$ | $\frac{3}{12}=0.25$ | $\frac{2}{12}=0.167$	B1	0 | 1 | 2 | 2 −1 | −1 | 0 | 1 | 1 0 | 0 | 1 | 2 | 2 1 | 1 | 2 | 3 | 3 Table with x values and at least one probability substituted, $0 < p < 1$. Condone any additional x values if probability stated as 0.
		B1	2 correct identified probabilities.
		B1	All probabilities correct (accept to 3sf). **SC** if less than 2 correct probabilities: **SC B1** 4 or 5 probabilities summing to one.
		3	
(b)	$\mathrm{E}(X) = -\frac{1}{12}+\frac{4}{12}+\frac{6}{12}+\frac{6}{12}\left[=\frac{15}{12}\right]$	M1	May be implied by use in Variance, accept unsimplified expression. Probabilities must sum to 1 ± 0.001.
	$\mathrm{Var}(X) = \frac{1}{12}+0+\frac{4}{12}+\frac{12}{12}+\frac{18}{12}-\left(\frac{15}{12}\right)^2$	M1	Appropriate variance formula using *their* $(\mathrm{E}(X))^2$. **FT** accept probabilities not summing to 1. Condone $\frac{35}{12}-\left(\frac{15}{12}\right)^2$ *or* $\frac{35}{12}-\frac{25}{9}$ from correct table.
	$\left[\frac{35}{12}-\frac{25}{16}=\right]\frac{65}{48}$, 1.35	A1	WWW
		3	

209. 9709_w21_ms_52 Q: 3

Question	Answer	Marks	Guidance
(a)	For one yellow: YGG + GYG +GGY $\frac{5}{9}\times\frac{4}{8}\times\frac{3}{7}\times 3$	M1	$\frac{a}{9}\times\frac{b}{8}\times\frac{c}{7}$, $0 < a,b,c$ integers $\leqslant 5$, for one arrangement.
		M1	*Their* three-factor probability × 3, ${}^3C_1$, ${}^3C_2$ or ${}^3P_1$, (or repeated adding) no additional terms.
	$\left[\frac{180}{504}=\right]\frac{5}{14}$	A1	AG. Convincingly shown, including identifying possible scenarios, may be on tree diagram WWW.
		3	
	Alternative method for question 3(a)		
	$\frac{{}^5C_1\times{}^4C_2}{{}^9C_3}$	M1	$\frac{{}^5C_1\times{}^4C_2}{{}^9C_r}$, $r = 2, 3, 4$
		M1	$\frac{{}^5C_s\times{}^4C_t}{{}^9C_3}$, $s+t=3$
	$\left[\frac{30}{84}=\right]\frac{5}{14}$	A1	AG. Convincingly shown, WWW.
		3	

Question	Answer	Marks	Guidance
(b)	X: 0 | 1 | 2 | 3 $P(X)$: $\frac{24}{504}$ $\left[=\frac{1}{21}, 0.0476\right]$ | $\frac{180}{504}$ $\left[=\frac{5}{14}, 0.357\right]$ | $\frac{240}{504}$ $\left[=\frac{10}{21}, 0.476\right]$ | $\frac{60}{504}$ $\left[=\frac{5}{42}, 0.119\right]$	**B1**	Table with correct X values and one correct probability inserted appropriately. Condone any additional X values if probability stated as 0.
		B1	Second identified correct probability, may not be in table.
		B1	All probabilities identified and correct . **SC** if less than 2 correct probabilities or X value(s) omitted: **SC B1** 3 or 4 probabilities summing to one.
		3	
(c)	[E(X) =] $\frac{840}{504}, \frac{5}{3}$, 1.67	**B1**	OE Must be evaluated. **SC B1 FT** correct unsimplified expression from incorrect **3(b)** using at least 3 probabilities, $0 < p < 1$.
		1	

210. 9709_w21_ms_52 Q: 5

Question	Answer	Marks	Guidance
(a)	[P(0, 1, 2) =] ${}^{10}C_0\, 0.16^0\, 0.84^{10} + {}^{10}C_1\, 0.16^1\, 0.84^9 + {}^{10}C_2\, 0.16^2\, 0.84^8$ [= 0.17490 + 0.333145 + 0.28555]	**M1**	One term: ${}^{10}C_x p^x (1-p)^{10-x}$ for $0 < x < 10$, any p.
		A1	Correct unsimplified expression, or better.
	0.794	**A1**	$0.7935 < p \leqslant 0.794$, mark at most accurate. If **M0** scored, **SC B1** for final answer 0.794.
		3	
(b)	$(0.84)^7\, 0.16$	**M1**	$(1-p)^7 p$, $0 < p < 1$
	0.0472	**A1**	0.0472144 to at least 3sf.
		2	

Question	Answer	Marks	Guidance
(c)	$4 \times 0.0472 \times (1 - 0.0472)^3$	**M1**	$4 \times q(1-q)^3$, q = *their* **(b)** or correct.
	0.163	**A1**	$0.163 \leqslant p \leqslant 0.1634$, mark at most accurate from *their* probability to at least 3sf.
		2	

211. 9709_w21_ms_53 Q: 6

Question	Answer	Marks	Guidance
(a)	$p+q+0.65=1$	**B1**	Sum of probabilities = 1.
	$p+2q+0.15=0.55$	**B1**	Use given information.
	Solve 2 linear equations	**M1**	Either a single expression with one variable eliminated formed or two expressions with both variables on the same side seen with at least one variable value stated.
	$p=0.3, \frac{3}{10}, \quad q=0.05, \frac{1}{20}$	**A1**	CAO, both WWW If **M0** with correct answers **SC B1**.
		4	
(b)	Var (X) = *their* $0.3+4\times$ *their* $0.05+9\times0.05-0.55^2$	**M1**	Appropriate variance formula including $(E(X))^2$, accept unsimplified.
	$0.6475 \left[\frac{259}{400}\right]$	**A1**	CAO (must be exact).
		2	
(c)	$1 - P(0, 1, 2) = 1 - ({}^{12}C_0\, 0.3^0\, 0.7^{12} + {}^{12}C_1\, 0.3^1\, 0.7^{11} + {}^{12}C_2\, 0.3^2\, 0.7^{10})$	**M1**	One correct term: ${}^{12}C_x p^x (1-p)^{12-x}$ for $0 < x < 12$, $0 < p < 1$.
	$1 - (0.01384 + 0.07118 + 0.16779)$	**A1FT**	Correct unsimplified expression, or better in final answer. Unsimplified expression must be seen to **FT** *their p* from **6(a)** or correct.
	0.747	**A1**	
		3	
(d)	$(0.95)^8 \times 0.05 = 0.0332$ or $0.95^8 - 0.95^9 = 0.0332$	**B1**	Evaluated.
		1	

212. 9709_m20_ms_52 Q: 2

Question	Answer	Marks	Guidance
(a)	$\left(\frac{1}{3}\right)\left(\frac{2}{3}\right)^2 + \left(\frac{1}{3}\right)\left(\frac{2}{3}\right)^3 + \left(\frac{1}{3}\right)\left(\frac{2}{3}\right)^4$	**M1**	One correct term with $0 < p < 1$
	$= \frac{4}{27} + \frac{8}{81} + \frac{16}{243} \left(= \frac{2432}{7776}\right)$	**A1**	Correct expression, accept unsimplified
	$= \frac{76}{243}$ or 0.313	**A1**	
		3	

Question	Answer	Marks	Guidance
(b)	x: 0, 1, 2, 3; P(x): $\frac{8}{27}$, $\frac{12}{27}$, $\frac{6}{27}$, $\frac{1}{27}$	B1	Probability distribution table with correct values of x, no additional values unless with probability of 0 stated, at least one non-zero probability included
	$P(0)=\left(\frac{2}{3}\right)^3$ $P(1)=\left(\frac{1}{3}\right)\left(\frac{2}{3}\right)^2\times 3$ $P(2)=\left(\frac{2}{3}\right)\left(\frac{1}{3}\right)^2\times 3$ $P(3)=\left(\frac{1}{3}\right)^3$	B1	1 correct probability seen (may not be in table) **or** 3 or 4 non-zero probabilities summing to 1
		B1	All probabilities correct
		3	
(c)	$E(X)=\left[0\times\frac{8}{27}\right]+1\times\frac{12}{27}+2\times\frac{6}{27}+3\times\frac{1}{27}$ $=\left[\frac{0}{27}\right]+\frac{12}{27}+\frac{12}{27}+\frac{3}{27}$	M1	Correct method from *their* probability distribution table with at least 3 terms, $0 \leqslant$ *their* P(x) $\leqslant 1$, accept unsimplified
	$=1$	A1	
		2	

213. 9709_s20_ms_51 Q: 1

Question	Answer	Marks
(a)	Prob of 4 (from 1,3, 3,1 or 2,2) $=\frac{3}{36}=\frac{1}{12}$ **AG**	B1
		1
(b)	Mean $=\frac{1}{\frac{1}{12}}=12$	B1
		1
(c)	$\left(\frac{11}{12}\right)^5\times\frac{1}{12}=0.0539$ or $\frac{161051}{2985984}$	B1
		1
(d)	$1-\left(\frac{11}{12}\right)^7$	M1
	0.456 or $\frac{16344637}{35831808}$	A1
		2

214. 9709_s20_ms_51 Q: 3

Question	Answer	Marks
(a)	x: 0, 1, 2, 3 Probability: $\frac{1}{56}$, $\frac{15}{56}$, $\frac{30}{56}$, $\frac{10}{56}$ (**B1** for probability distribution table with correct outcome values)	**B1**
	$P(0)=\frac{3}{8}\times\frac{2}{7}\times\frac{1}{6}=\frac{1}{56}$ $P(1)=\frac{5}{8}\times\frac{3}{7}\times\frac{2}{6}\times3=\frac{15}{56}$ $P(2)=\frac{5}{8}\times\frac{4}{7}\times\frac{3}{6}\times3=\frac{30}{56}$ $P(3)=\frac{5}{8}\times\frac{4}{7}\times\frac{3}{6}=\frac{10}{56}$ (**M1** for denominator $8\times7\times6$)	**M1**
	Any one probability correct (with correct outcome)	**A1**
	All probabilities correct	**A1**
		4
(b)	$1-P(8, 9, 10)=1-\left[{}^{10}C_8\,0.64^8 0.36^2+{}^{10}C_9\,0.64^9 0.36^1+0.64^{10}\right]$	**M1**
	$1-(0.164156+0.064852+0.11529)$	**M1**
	0.759	**A1**
		3

215. 9709_s20_ms_52 Q: 5

Question	Answer	Marks
(a)	| | 1 | 1 | 2 | 2 | 3 | | 1 | 1 | 1 | 2 | 2 | 3 | | 2 | 2 | 2 | 2 | 2 | 3 | | 3 | 3 | 3 | 3 | 3 | 3 |	**M1**
	$\frac{7}{15}$ **AG**	**A1**
		2
(b)	x: 1, 2, 3 Probability: $\frac{2}{15}$, $\frac{6}{15}$, $\frac{7}{15}$	**B1**
	P(1) or P(2) correct	**B1**
	3rd probability correct, **FT** sum to 1	**B1**
		3

Question	Answer	Marks
(c)	$E(X) = \frac{2+12+21}{15} = \frac{35}{15} = \frac{7}{3}$	B1
	$\text{Var}(X) = \frac{1^2\times2+2^2\times6+3^2\times7}{15} - \left(\frac{7}{3}\right)^2$	M1
	$\frac{22}{45}(0.489)$	A1
		3

216. 9709_s20_ms_53 Q: 2

Question	Answer	Marks
(a)	$0.22^3 = 0.0106$	B1
		1
(b)	$P(2, 3, 4) = {}^{16}C_2\ 0.22^2 0.78^{14} + {}^{16}C_3 0.22^3 0.78^{13} + {}^{16}C_4 0.22^4 0.78^{12}$	M1
	$0.179205 + 0.235877 + 0.216221$	A1
	0.631	A1
		3

217. 9709_s20_ms_53 Q: 4

Question	Answer	Marks
(a)	<table><tr><td>−1</td><td>0</td><td>0</td><td>1</td></tr><tr><td>0</td><td>1</td><td>1</td><td>2</td></tr><tr><td>2</td><td>3</td><td>3</td><td>4</td></tr></table>	
	<table><tr><td>x</td><td>−1</td><td>0</td><td>1</td><td>2</td><td>3</td><td>4</td></tr><tr><td>Probability</td><td>$\frac{1}{12}$</td><td>$\frac{3}{12}$</td><td>$\frac{3}{12}$</td><td>$\frac{2}{12}$</td><td>$\frac{2}{12}$</td><td>$\frac{1}{12}$</td></tr></table>	
	Probability distribution table with correct scores with at least one probability	B1
	At least 4 probabilities correct	B1
	All probabilities correct	B1
		3
(b)	$E(X) = \frac{-1+0+3+4+6+4}{12} = \frac{16}{12} = \frac{4}{3}$	B1
	$\text{Var}(X) = \frac{1+0+3+8+18+16}{12} - \left(\frac{4}{3}\right)^2$	M1
	$\frac{37}{18}$ (= 2.06)	A1
		3

218. 9709_w20_ms_51 Q: 3

(a)	$P(X > 6) = 0.75^6$	**M1**	p^n, $n = 6, 7$ $0 < p < 1$
	0.178, $\frac{729}{4096}$	**A1**	0·17797…
		2	

Question	Answer	Marks	Guidance
(b)	$1 - P(0, 1, 2) = 1 - (0.75^{10} + {}^{10}C_1\ 0.25^1 0.75^9 + {}^{10}C_2 0.25^2 0.75^8)$	**M1**	Binomial term of form ${}^{10}C_x\ p^x (1-p)^{10-x}$, $0 < p < 1$, any p, $x \neq 0, 10$
	1 – (0·0563135 + 0·1877117 + 0·2815676)	**A1**	Correct unsimplified expression
	0·474	**A1**	$0{\cdot}474 \leqslant p \leqslant 0{\cdot}4744$
		3	

219. 9709_w20_ms_51 Q: 4

(a)

y	1	2	3	4
prob	$\frac{7}{16}$	$\frac{5}{16}$	$\frac{3}{16}$	$\frac{1}{16}$

B1

	1	2	3	4
1	1	1	2	3
2	1	2	1	2
3	2	1	3	1
4	3	2	1	4

Probability distribution table with correct scores with at least one probability, allow extra score values if probability of zero stated'

B1 One probability (linked with correct score) correct

B1 2 more probs (linked with correct scores) correct

B1 FT 4th prob correct, FT sum of 3 or 4 terms = 1

4

Question	Answer	Marks	Guidance
(b)	$P(2|\text{even}) = \frac{\frac{5}{16}}{\frac{6}{16}}$	**M1**	$\frac{\textit{their}\ P(2)}{\textit{their}\ P(2) + \textit{their}\ P(4)}$ seen or correct outcome space.
	$\frac{5}{6}$ or 0·833	**A1**	
		2	

220. 9709_w20_ms_52 Q: 1

Question	Answer	Marks	Guidance
(a)	$1-\left(\frac{5}{6}\right)^5$ or $\frac{1}{6}+\frac{5}{6}\times\frac{1}{6}+\left(\frac{5}{6}\right)^2\times\frac{1}{6}+\left(\frac{5}{6}\right)^3\times\frac{1}{6}+\left(\frac{5}{6}\right)^4\times\frac{1}{6}$	M1	$1-p^n$ $n=5,6$ or $p+pq+pq^2+pq^3+pq^4$ $(+pq^5)$ $0<p<1$, $p+q=1$,
	0·598, $\frac{4651}{7776}$	A1	
		2	
(b)	(1 – P(0, 1, 2)) $1-\left(\left(\frac{5}{6}\right)^{10}+{}^{10}C_1\left(\frac{1}{6}\right)\left(\frac{5}{6}\right)^9+{}^{10}C_2\left(\frac{1}{6}\right)^2\left(\frac{5}{6}\right)^8\right)$	M1	${}^{10}C_x\ p^x(1-p)^{10-x}$, $0<p<1$, any p, $x\neq 0,10$
	1 – (0·1615056 + 0·3230111 + 0·290710)	A1	Correct expression, accept unsimplified, condone omission of final bracket
	0·225	A1	$0{\cdot}2247<p\leq 0{\cdot}225$, WWW
		3	

221. 9709_w20_ms_52 Q: 2

Question	Answer	Marks	Guidance
(a)	P(1 red) = $\frac{5}{8}\times\frac{3}{7}\times\frac{2}{6}\times 3$	M1	$\frac{a}{8}\times\frac{b}{7}\times\frac{c}{6}\times k$ or $\frac{5}{d}\times\frac{3}{e}\times\frac{2}{f}\times 3$, $1\leqslant a,b,c\leqslant 5$, $d,e,f\leqslant 8$, a, b, c, d, e, f, k all integers. $1<k\leqslant 3$,
	$\frac{15}{56}$	A1	AG, WWW
	Alternative method for question 2(a)		
	$\frac{{}^5C_1\times{}^3C_2}{{}^8C_3}$	M1	$\frac{{}^aC_1\times{}^bC_2}{{}^8C_3}$ or $\frac{{}^5C_d\times{}^3C_e}{{}^8C_3}$ or $\frac{{}^5C_d\times{}^3C_e\ (or\ {}^aC_1\times{}^bC_2)}{{}^5C_3\times{}^3C_0+{}^5C_2\times{}^3C_1+{}^5C_1\times{}^3C_2+{}^5C_0\times{}^3C_3}$, $a+b=8$, $d+e=3$
	$\frac{15}{56}$	A1	AG, WWW, $\frac{15}{56}$ must be seen
		2	
(b)	x: 0 | 1 | 2 | 3 Prob.: $\frac{1}{56}$ | $\frac{15}{56}$ | $\frac{30}{56}=\frac{15}{28}$ | $\frac{10}{56}=\frac{5}{28}$ 0·0179 | 0·268 | 0·536 | 0·179	B1	Probability distribution table with correct outcomes with at least one probability less than 1, allow extra outcome values if probability of zero stated.
		B1	2 of P(0), P(2) and P(3) correct
		B1 FT	4th probability correct or FT sum of 3 or more probabilities = 1, with P(1) correct
		3	

Question	Answer	Marks	Guidance
(c)	$\text{Var}(X)=\frac{(0^2\times 1)+1^2\times 15+2^2\times 30+3^2\times 10}{56}-\left(\frac{15}{8}\right)^2$ $=\frac{15}{56}+\frac{120}{56}+\frac{90}{56}-\left(\frac{15}{8}\right)^2$	M1	Substitute *their* attempts at scores in correct variance formula, must have '$-\text{mean}^2$' (FT if mean calculated) (condone probabilities not summing to 1 for this mark)
	$\frac{225}{448}$, 0·502	A1	
		2	

222. 9709_w20_ms_53 Q: 2

Question	Answer	Marks	Guidance
(a)	$\left(\frac{5}{6}\right)^8$	M1	p^8, $0 < p < 1$, no x, + or -
	0.233	A1	
		2	
(b)	36	B1	
		1	
(c)	$P(X=10) + P(X=11) = \left(\frac{35}{36}\right)^9 \frac{1}{36} + \left(\frac{35}{36}\right)^{10} \frac{1}{36}$	M1	OE, unsimplified expression in form $p^9q + p^{10}q$, $p + q = 1$, no ×
	0.0425	A1	
		2	

223. 9709_w20_ms_53 Q: 6

Question	Answer	Marks	Guidance
(a)	Scenarios: HHT: $\frac{2}{3}\times\frac{2}{3}\times\frac{1}{5}=\frac{4}{45}$ HTH: $\frac{2}{3}\times\frac{1}{3}\times\frac{4}{5}=\frac{8}{45}$ THH: $\frac{1}{3}\times\frac{2}{3}\times\frac{4}{5}=\frac{8}{45}$	M1	One 3 factor probability with 3, 3, 5 as denominators
		M1	3 factor probabilities for 2 or 3 correct scenarios added, no incorrect scenarios
	Total = $\frac{20}{45}=\frac{4}{9}$	A1	AG, Total of 3 products with clear context
		3	
(b)	x: 0, 1, 2, 3 Prob.: $\frac{1}{45}$, $\frac{8}{45}$, $\frac{20}{45}$, $\frac{16}{45}$	B1	Probability distribution table with correct outcomes with at least one probability, allow extra outcome values if probability of zero stated'
		B1	2 of P(0), P(1) and P(3) correct
		B1 FT	3 or 4 probabilities sum to 1 with P(2) correct
		3	
(c)	$\text{Var(X)} = \frac{0^2\times1+1^2\times8+2^2\times20+3^2\times16}{45} - \left(\frac{32}{15}\right)^2$ $= \frac{8}{45}+\frac{80}{45}+\frac{144}{45}-\left(\frac{32}{15}\right)^2$	M1	Substitute *their* attempts at scores in correct variance formula, must have '– mean^2' (FT if calculated) (condone probs not summing to 1); must be at least 2 non-zero values
	$\frac{136}{225}$ or 0.604	A1	
		2	

224. 9709_m19_ms_62 Q: 4

Question	Answer	Marks	Guidance
(i)	x | -1 | 1 | 2 | 3 p | k | k | $4k$ | $9k$	B1	Probability distribution table with correct values of x, no additional values unless with probability 0 stated, at least one correct probability including k
	$15k = 1$,	M1	Equating $\Sigma p = 1$, may be implied by answer
	$k = \frac{1}{15}$	A1	If 0 scored, SCB2 for probability distribution table with correct numerical probabilities.
		3	

Question	Answer	Marks	Guidance
(ii)	**Method 1**		
	$E(X) = 8k + 27k = 35k = \frac{35}{15} = \frac{7}{3}$	B1FT	FT if 0< *their k*<1
	$\text{Var}(X) = (k + k + 16k + 81k) - (35k)^2$	M1	Correct formula for variance, in terms of k at least – must have '– mean^2'(ft).
	$= 1.16$, $\frac{52}{45}$	A1	
	Method 2		
	$E(X) = \frac{8}{15} + \frac{27}{15} = \frac{35}{15} = \frac{7}{3}$	B1FT	FT if 0< *their k*<1
	$\text{Var}(X) = \frac{1}{15} + \frac{1}{15} + \frac{16}{15} + \frac{81}{15} - \left(\frac{7}{3}\right)^2$	M1	Subst *their* values in correct var formula – must have '– mean^2'(ft) (condone probs not summing to exactly 1)
	= 1.16 (= 52/45)	A1	Using their values from **(i)**
		3	

225. 9709_s19_ms_61 Q: 6

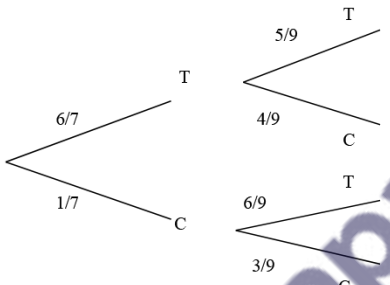
Question	Answer	Marks	Guidance
(i)	P(loses \$1) = P(F and F) = 0.8 × 0.8	M1	0.8 x 0.8 or (1 – 0.2)(1-0.2) or P(F) × P(F) or P(F)+P(F) seen or implied
	= 0.64 AG	A1	Must see probabilities multiplied together with final answer and a clear probability statement or implied by labelled tree diagram
		2	

Question	Answer	Marks	Guidance
(ii)	Amount gained (\$) | -1 | 0.50 | 2 Prob | | 0.16 | 0.2	B1	–1 linked with 0.64 in table
		B1	0.5 seen in table
		B1	0.16 seen in table linked to their 0.5
		B1	**FT** P(2.00 gained) = 0.36 – P(0.50 gained) or correct, and all amount gained linked correctly in table
		4	
(iii)	E(winnings) = –1 × 0.64 + 0.5 × 0.16 + 2 × 0.2 = –(\$)0.16, –16 cents	B1	**FT** Accept (\$)0.16 or 16 cents **loss.** FT unsimplified E(winnings) from their table provided $\Sigma p = 1$
		1	

226. 9709_s19_ms_62 Q: 3

Question	Answer	Marks	Guidance
(i)	P(at most 7) = 1 – P(8, 9, 10) = 1 – ^{10}C8(0.35)^{8}(0.65)^{2} – ^{10}C$_{9}$(0.35)^{9}(0.65)^{1} – (0.35)^{10}	**M1**	Use of normal approximation M0 Binomial term of form $^{10}C_x p^x(1-p)^{10-x}$ $0 < p < 1$ any p, $x \neq 10,0$
	[= 1 – 0.004281 – 0.0005123 – 0.00002759]	**A1**	Correct unsimplified (or individual terms evaluated) answer seen Condone 1 – A + B + C leading to correct solution
	= 0.995	**B1**	B1 **not** dependent on previous marks.
	Alternative method for question 3(i)		
	P(at most 7) = P(0,1,2,3,4,5,6,7)	**M1**	Binomial term of form $^{10}C_x p^x(1-p)^{10-x}$ $0 < p < 1$ any p, $x \neq 10,0$
	= (0.65)^{10} + ^{10}C1(0.35)^{1}(0.65)^{9}+…+ ^{10}C$_{7}$(0.35)^{7}(0.65)^{3}	**A1**	Correct unsimplified answer or individual terms evaluated seen
	= 0.995	**B1**	
		3	
(ii)	1 – (0.65)^{n} > 0.99 0.01 > (0.65)^{n}	**M1**	Equation or inequality with (0.65)^{n} **and** 0.01 or (0.35)^{n} **and** 0.99 only (Note 1 – 0.99 is equivalent to 0.01 etc.)
	n > 10.69	**M1**	Solving their $a^n = c$, $0 < a,c < 1$ using logs **or** Trial and Error If answer inappropriate, at least 2 trials are required for Trial and Error M mark
	smallest n = 11	**A1**	CAO
		3	

227. 9709_s19_ms_62 Q: 5

Question	Answer	Marks	Guidance
(i)		**B1**	First pair of branches labels and probs correct (6/7 and 1/7 or rounding to 0.857 and 0.143) (Labelling must be logically…e.g. (T and T) or (T and Not T) would be acceptable)
		B1	Either of second top pair or bottom of branches labels and probs correct
		B1	Both second pairs of branches labels and probs correct. No additional / further branches.
		3	
(ii)	No of toffees taken (T): 0 | 1 | 2 prob: $\frac{3}{63}$, 0.0476(2) | $\frac{30}{63}$, 0.476(2) | $\frac{30}{63}$, 0.476(2)	**B1**	P(1) correct
		B1	P(0) or P(2) correct
		B1	**FT** Correct values in table, any additional values of T have stated probability of zero. For FT $\Sigma p = 1$,
		3	
(iii)	E(X) = $\frac{90}{63}$ ($\frac{10}{7}$) (1.43)	**B1**	Not FT
		1	

Question	Answer	Marks	Guidance
(iv)	$P(1^{st}\ C \mid 2^{nd}\ T) = \frac{P(C \cap T)}{P(T)} = \frac{\frac{1}{7}\times\frac{6}{9}}{\frac{1}{7}\times\frac{6}{9}+\frac{6}{7}\times\frac{5}{9}} = \frac{\frac{6}{63}}{\frac{36}{63}}$	B1	P(C∩T) attempt seen as numerator of a fraction, consistent with *their* tree diagram or correct
		M1	Summing 2 appropriate two-factor probabilities, consistent with *their* tree diagram or correct seen anywhere
		A1	$\frac{36}{63}$ oe or correct unsimplifed expression seen as numerator or denominator of a fraction
	$\frac{1}{6}$ oe	A1	Final answer
		4	

228. 9709_s19_ms_63 Q: 6

Question	Answer	Marks	Guidance
(i)	score: 1, 2, 3, 4, 6, 9; prob: $\frac{3}{15}$, $\frac{4}{15}$, $\frac{4}{15}$, $\frac{1}{15}$, $\frac{2}{15}$, $\frac{1}{15}$	B1	Probability distribution table with correct scores, allow extra score values if probability of zero stated
		B1	2 probabilities (with correct score) correct
		B1	3 or more correct probabilities with correct scores
		B1	FT $\Sigma p = 1$, at least 4 probabilities
		4	
(ii)	$\text{mean} = \frac{(3+8+12+4+12+9)}{15} = \frac{48}{15}$ (3.2)	B1	
	$\text{Var} = \frac{(3+16+36+16+72+81)}{15} - (\textit{their}\ 3.2)^2$	M1	FT Substitute *their* attempts at scores in correct var formula, must have "– mean^2" (condone probabilities not summing to 1)
	$= \frac{224}{15} - 3.2^2 = 4.69\ \left(\frac{352}{75}\right)$	A1	
		3	
(iii)	Score of 4, 6, 9	M1	Identifying relevant scores from *their* mean and *their* table
	Prob $\frac{4}{15}$ (0.267)	A1	Correct answer **SC** B1 for 4/15 with no working
		2	

229. 9709_w19_ms_61 Q: 2

Question	Answer	Marks	Guidance
(i)	$1-({}^{10}C_2\,0.42^8\,0.58^2 + {}^{10}C_9\,0.42^9\,0.58^1 + 0.42^{10})$	M1	Binomial term of form ${}^{10}C_a p^a (1-p)^b$ $0<p<1$ any p, $0 \leqslant a,b \leqslant 10$
		A1	Correct unsimplified expression
	0.983	A1	
		3	
(ii)	$1 - P(0) > 0.995$ $0.58^n < 0.005$	M1	Equation or inequality involving 0.58^n or 0.42^n and 0.995 or 0.005
	$n > \frac{\log 0.005}{\log 0.58}$ $n > 9.727$	M1	Attempt to solve using logs or Trial and Error. May be implied by their answer (rounded or truncated)
	$n = 10$	A1	CAO
		3	

230. 9709_w19_ms_61 Q: 4

Question	Answer	Marks	Guidance
(i)	$\frac{1}{4}++p+p+\frac{3}{8}+4p=1$	M1	Unsimplified sum of probabilities equated to 1
	$p=\frac{1}{16}$	A1	If method FT from *their* incorrect **(i)**, expressions for E(X) and Var(X) must be seen unsimplified with all probabilities <1, condone not adding to 1
		2	

Question	Answer	Marks	Guidance
(ii)	$[\mathrm{E}(X)]=-\frac{1}{4}+\frac{1}{16}+\frac{6}{8}+1=\frac{25}{16}$	M1	May be implied by use in Variance, accept unsimplified
	$[\mathrm{Var}(X)]=\frac{1}{4}+\frac{1}{16}+\frac{12}{8}+\frac{16}{4}-\left(\textit{their}\,\frac{25}{16}\right)^2$	M1	Substitute into correct variance formula, must have '– their mean^2'
	$\frac{863}{256}$ or 3.37	A1	OE
		3	
(iii)	$\mathrm{P}(X=2\|X>0)=\frac{\mathrm{P}(X=2)}{\mathrm{P}(X>0)}=\frac{\frac{3}{8}}{\frac{11}{16}}$	M1	Conditional probability formula used consistent with their probabilities
	$\frac{6}{11}$ or 0.545	A1	
		2	

231. 9709_w19_ms_62 Q: 5

Question	Answer	Marks	Guidance
(i)	x: −1, 0, 1, 2, 3, 4 p: $\frac{1}{12}$, $\frac{1}{12}$, $\frac{3}{12}$, $\frac{2}{12}$, $\frac{3}{12}$, $\frac{2}{12}$	B1	Table with correct values of x, at least 1 probability, all probabilities $\leqslant 1$
		B1	2 probabilities correct, may not be in table
		B1	2 more probabilities correct, may not be in table
		B1	All correct, values in table SC1 No more than 1 correct probability and at least 5 probabilities summing to 1 in table
		4	
(ii)	$[\mathrm{E}(X)]=\left(\frac{-1+0+3+4+9+8}{12}\right)=\frac{23}{12}$	M1	May be implied by use in variance. Allow unsimplified expression
	$[\mathrm{Var}(X)]=\frac{1+0+3+8+27+32(=71)}{12}-\left(\frac{23}{12}\right)^2$	M1	Appropriate variance formula using *their* E(X)^2
	2.24 or $\frac{323}{144}$ or $2\frac{35}{144}$	A1	CAO
		3	

232. 9709_w19_ms_63 Q: 6

Question	Answer	Marks	Guidance
(i)	$P(RR) = \frac{3}{8} \times \frac{2}{7} = \frac{3}{28}$	**B1**	OE
		1	
(ii)	P(RW) + P(WR) $\frac{3}{8} \times \frac{5}{7} + \frac{5}{8} \times \frac{3}{7}$	**M1**	Method shown, numerical calculations identified, may include replacements
	$= \frac{15}{28}$	**A1**	AG, Fully correct calculations
	Alternative method for question 6(ii)		
	1 – (P(RR) + P(WW)) $1 - \left(\frac{3}{28} + \frac{5}{8} \times \frac{4}{7}\right)$	**M1**	Method shown, numerical calculations identified, may include replacements
	$= \frac{15}{28}$	**A1**	AG, Fully correct calculations
		2	
(iii)	P(first red|second red) = $\frac{\textit{their}\ \textbf{(i)}}{\textit{their}\ \textbf{(i)} + \frac{5}{8} \times \frac{3}{7}} = \frac{\frac{3}{8} \times \frac{2}{7}}{\frac{3}{8} \times \frac{2}{7} + \frac{5}{8} \times \frac{3}{7}} = \frac{\frac{3}{28}}{\frac{21}{56}}$	**M1**	Conditional probability formula used consistent with *their* probabilities or correct
	$= \frac{2}{7}$	**A1**	OE
		2	

Question	Answer	Marks	Guidance
(iv)	x | 0 | 1 | 2 p | $\frac{10}{28}$ | $\frac{15}{28}$ | $\frac{3}{28}$	**B1**	Probability distribution table with correct values of x and at least one correct probability placed. Extra x values allowed with probability of zero stated.
		B1FT	Fully correct FT P(2) = *their* **(i)**, P(1) = *their* **(ii)**, $\Sigma(p) = 1$.
		2	
(v)	$E(X) = \frac{30}{56} + \frac{12}{56} = \frac{42}{46}$ $\left(= \frac{3}{4}\right)$	**B1**	May be implied by use in variance formula
	$\text{Var}(X) = \frac{30}{56} + \frac{24}{56} - \left(\textit{their}\ \frac{3}{4}\right)^2$	**M1**	Substitute into correct variance formula, must have '$-$ *their* mean^2' Must be for 2 or more non-zero x-values
	$\frac{45}{112}$ or 0.402	**A1**	Correct final answer
		3	

233. 9709_m18_ms_62 Q: 4

Question	Answer	Marks	Guidance
(i)	$\Sigma p = 1$: $0.2 + 0.1 + p + 0.1 + q = 1$: $p + q = 0.6$	**M1**	Unsimplified sum of probabilities equated to 1
	$\Sigma px = 1.7$: $-0.4 + 0 + p + 0.3 + 4q = 1.7$:	**M1**	Unsimplified Sum of px equated to 1.7
	$p + 4q = 1.8$	**M1**	Solve simult. equations to find expression in p or q
	$p = 0.2$, $q = 0.4$	**A1**	
		4	
(ii)	$\text{Var}(X) = \Sigma px^2 - 1.7^2 = 4\text{x}0.2 + 1p + 9\text{x}0.1 + 16q - 1.7^2$ $= 8.3 - 2.89$	**M1**	Use correct unsimplified expression for variance
	$= 5.41$	**A1**	
		2	

234. 9709_s18_ms_61 Q: 3

Question	Answer	Marks	Guidance
(i)	$\text{P(RB)} + \text{P(BR)} = \frac{4}{12} \times \frac{8}{11} + \frac{8}{12} \times \frac{4}{11}$ oe	**M1**	Multiply 2 probs together and summing two 2-factor probs, unsimplified, condone replacement
	$\text{P(diff colours)} = \frac{64}{132}\ (\frac{16}{33})\ (0.485)$ oe	**A1**	Correct answer
	Method 2 $1 - \text{P(BB)} - \text{P(RR)} = 1 - \frac{4}{12} \times \frac{3}{11} - \frac{8}{12} \times \frac{7}{11}$	**M1**	Multiply 2 probs together and subtracting two 2-factor probs from 1, unsimplified, condone replacement
	$\text{P(diff colours)} = \frac{64}{132}\ (\frac{16}{33})$ oe	**A1**	Correct answer
	Method 3 $\text{P(diff colours)} = \frac{({}^4C_1 \times {}^8C_1)}{{}^{12}C_2}$	**M1**	Multiply 2 combs together and dividing by a combination
	$= \frac{16}{33}$	**A1**	Correct answer
		2	
(ii)	Number of red socks: 0 | 1 | 2 Prob: $\frac{14}{33}$ | $\frac{16}{33}$ | $\frac{3}{33}$	**B1**	Prob distribution table drawn, top row correct, condone additional values with $p = 0$ stated
		B1	P(0) or P(2) correct to 3sf (need not be in table)
		B1	All probs correct to 3sf, condone P(0) and P(2) swapped if correct
		3	

Question	Answer	Marks	Guidance
(iii)	$\text{E}(X) = 1 \times \frac{16}{33} + 2 \times \frac{3}{33} = \frac{16}{33} + \frac{6}{33} = \frac{22}{33}\ (\frac{2}{3})$	**B1ft**	ft their table if 0, 1, 2 only, $0 < p < 1$
		1	

235. 9709_s18_ms_61 Q: 6

Question	Answer	Marks	Guidance
(i)	P(SLL) = (0.3)(0.55)(0.55) = 0.09075 ($\frac{363}{4000}$)	M1	P(SLL), P(SRR), P(SSL) or P(SSR) seen
	P(SRR) = (0.3)(0.15)(0.15) = 0.00675 ($\frac{27}{4000}$)	A1	Two correct options 0.09075 or 0.00675 can be unsimplified
	Total = ${}^3C_1 \times$ P(SLL) + ${}^3C_1 \times$ P(SRR) = 0.27225 + 0.02025	M1	Summing 6 prob options not all identical
	Prob = 0.293 accept 0.2925 ($\frac{117}{400}$)	A1	Correct answer
		4	
(ii)	$P(\text{SSS} \mid \text{all same dir}^n) = \frac{P(SSS \text{ and same dir}^n)}{P(\text{same direction})}$	B1	$(0.3)^3$ oe seen on its own as num or denom of a fraction
		M1	Attempt at P(*SSS*+*LLL*+*RRR*) seen anywhere
	$= \frac{0.3\times0.3\times0.3}{(0.15)^3+(0.55)^3+(0.3)^3}$	A1	$(0.15)^3 + (0.55)^3 + (0.3)^3$ oe seen as denom of a fraction
	= 0.137 ($\frac{108}{787}$)	A1	Correct answer
		4	

236. 9709_s18_ms_62 Q: 4

Question	Answer	Marks	Guidance
(i)	<table><tr><td>*X*</td><td>0</td><td>1</td><td>2</td></tr><tr><td>Prob</td><td>$\frac{2}{7}$</td><td>$\frac{4}{7}$</td><td>$\frac{1}{7}$</td></tr></table>	B1	Prob distribution table drawn, top row correct with at least one probability $0 < p < 1$ entered, condone additional values with $p = 0$ stated
	$P(0) = \frac{5}{7}\times\frac{4}{6}\times\frac{3}{5} = \frac{2}{7}$ (0.2857)	B1	One probability correct (need not be in table)
	$P(1) = \frac{2}{7}\times\frac{5}{6}\times\frac{4}{5}\times {}^3C_1 = \frac{4}{7}$ (0.5713)	B1	Another probability correct (need not be in table).
	$P(2) = \frac{2}{7}\times\frac{1}{6}\times\frac{5}{5}\times {}^3C_2 = \frac{1}{7}$ (0.1429)	B1	Values in table, all probs correct (to 3SF) or 3 probabilities summing to 1
		4	
(ii)	Var (*X*) $= 1\times\frac{4}{7} + 4\times\frac{1}{7} - (\frac{6}{7})^2$ $= \frac{8}{7} - (\frac{6}{7})^2$	M1	Unsimplified correct numerical expression for variance **or** *their* probabilities from **(i)** $0 < p < 1$ in **unsimplified** variance expression
	$= \frac{20}{49}$ or 0.408	A1	Correct answer (0.40816…) nfww Final answer does **not** imply the method mark
		2	

237. 9709_s18_ms_63 Q: 2

Question	Answer	Marks	Guidance
(i)	$z = 0.674$	B1	z value ±0.674
	$0.674 = \frac{0 - -3}{\sigma}$	M1	±Standardising with 0 and equating to a z-value
	$\sigma = 4.45$	A1	Correct answer www ie not ignoring a minus sign
	Total:	3	
(ii)	P(0, 1)	M1	Any bin of form ${}^8C_x(0.75)^x\,(0.25)^{8-x}$ any x
	$= (0.75)^8 + {}^8C_1(0.25)(0.75)^7$	M1	Correct unsimplified answer, may be implied by numerical values
	0.1001+ 0.2670 = 0.367	A1	Correct answer
	Method 2 $1 - P(8,7,6,5,4,3,2) = 1 - (0.25)^8 - {}^8C_1(0.75)(0.25)^7 - \ldots$	M1	Any bin of form ${}^8C_x(0.75)^x\,(0.25)^{8-x}$ any x
	$- {}^8C_2(0.75)^6\,(0.25)^2$	M1	Correct unsimplified answer
	$= 0.367$	A1	Correct answer
	Total:	3	

238. 9709_s18_ms_63 Q: 5

Question	Answer	Marks	Guidance
(i)	$P(0) = 0.6 \times 0.25 \times 0.5 = 0.075$ $P(1) = 0.4 \times 0.25 \times 0.5 + 0.6 \times 0.75 \times 0.5 + 0.6 \times 0.25 \times 0.5 = 0.35$ $P(2) = 0.4 \times 0.75 \times 0.5 + 0.4 \times 0.25 \times 0.5 + 0.6 \times 0.75 \times 0.5 = 0.425$ $P(3) = 0.4 \times 0.75 \times 0.5 = 0.15$	B1	0, 1, 2, 3 seen as top line of a pdf table OR attempting to evaluate P(0), P(1), P(2) and P(3)
		M1	Multiply 3 probabilities together from 0.4 or 0.6, 0.25 or 0.75, 0.5 with or without a table
		M1	Summing 3 probabilities for P(1) or P(2) with or without a table
		B1	One correct probability seen.
		A1	All correct in a table
	Total:	5	
(ii)	$E(X) = 0.35 + 2 \times 0.425 + 3 \times 0.15 = 1.65$ $\left(\frac{33}{20}\right.$ oe$\left.\right)$	M1	Correct unsimplified expression for the mean using their table, $\sum p = 1$; can be implied by correct answer
(ii)	$\text{Var}(X) = 0.35 + 4 \times 0.425 + 9 \times 0.15 - 1.65^2$	M1	Correct unsimplified expression for the variance using their table and their mean^2 subtracted, $\sum p = 1$
	$= 0.678\ (0.6775)$ $\left(\frac{271}{400}\right.$ oe$\left.\right)$	A1	Correct answer
	Total:	3	

No of heads	0	1	2	3
Prob	0.075 $\left(\frac{3}{40}\right)$	0.35 $\left(\frac{7}{20}\right)$	0.425 $\left(\frac{17}{40}\right)$	0.15 $\left(\frac{3}{20}\right)$

239. 9709_w18_ms_61 Q: 2

Question	Answer	Marks	Guidance
(i)	$6p + 0.1 = 1$ $p = 0.15$	B1	Correct answer
		1	
(ii)	$\text{Var}(X) = 1 \times p + 1 \times 2p + 4 \times 2p + 16 \times 0.1 - 1.15^2$	M1	Correct unsimplified formula, *their p* substituted (allow 1 error)
	$0.15 + 0 + 0.3 + 1.2 + 1.6 - 1.15^2$ $= 1.9275 = 1.93$ (3sf)	A1	Correct answer
		2	

240. 9709_w18_ms_62 Q: 3

Question	Answer	Marks	Guidance
(i)	**Method 1**		
	P(3) + P(4) + P(5) = ${}^5C_3\ 0.75^3 \times 0.25^2$ +	**M1**	One binomial term ${}^5C_x p^x(1-p)^{5-x}$, $x \neq 0$ or 5, any p
	${}^5C_4\ 0.75^4 \times 0.25^1$ + ${}^5C_5\ 0.75^5 \times 0.25^0$	**M1**	Correct unsimplified expression
	= 0.26367 + 0.39551 + 0.23730 = 0.896 (459/512)	**A1**	Correct final answer, allow 0.8965 (isw) but not 0.897 alone
	Method 2		
	1 – P(0) – P(1) – P(2) = 1 – ${}^5C_0\ 0.75^0 \times 0.25^5$	**M1**	One binomial term ${}^5C_x p^x(1-p)^{5-x}$, $x \neq 0$ or 5, any p
	– ${}^5C_1\ 0.75^1 \times 0.25^4$ – ${}^5C_2\ 0.75^2 \times 0.25^3$	**M1**	Correct simplified expression
	= 1 – 0.00097656 – 0.014648 – 0.087891 = 0.896 (459/512)	**A1**	Correct final answer, allow 0.8965 (isw) but not 0.897 alone
		3	

Question	Answer	Marks	Guidance
(ii)	**Method 1**		
	P(C,C) + P(C,C′) + P(C′,C) 0.8 × 0.9	**B1**	Unsimplified prob completed on both days
	0.8 × 0.1 + 0.2 × 0.6	**M1**	Unsimplified prob 0.8 × a + 0.2 × b, a = 0.1or 0.4, b = 0.6 or 0.9
	= 0.92 oe	**A1**	Correct final answer
	Method 2		
	1 – P(C′,C′) = 1 – 0.2 × 0.4	**B1**	Unsimplified prob completed on no days
		M1	1 – 0.2 × a, a=0.1or 0.4 allow unsimplified
	= 0.92	**A1**	Correct final answer
		3	

$P(S,S') = \frac{4}{11} \times \frac{7}{10} = \frac{28}{110}$

$P(P,P') = \frac{2}{11} \times \frac{9}{10} = \frac{18}{110}$

$P(I,I') = \frac{4}{11} \times \frac{7}{10} = \frac{28}{110}$

$P(M,M') = \frac{1}{11} \times \frac{10}{10} = \frac{10}{110}$

$Total = \frac{84}{110}$

$P(Same) = 1 - \frac{84}{110} = \frac{26}{110}$

B1 one of products correct
M1 1 – sum of probabilities from 4 appropriate scenarios
A1 Correct final answer

241. 9709_w18_ms_62 Q: 6

Question	Answer	Marks	Guidance
(i)	x: −2, −1, 0, 1, 2, 3 ; p: $\frac{1}{12}$, $\frac{2}{12}$, $\frac{3}{12}$, $\frac{3}{12}$, $\frac{2}{12}$, $\frac{1}{12}$	B1	−2, −1, 0, 1, 2, 3 seen as top line of a pdf table with at least 1 probability OR attempting to evaluate P(−2), P(−1), P(0), P(1), P(2), P(3) (condone additional values with p=0 stated)
		B1	At least 4 probs correct (need not be in table)
		B1	All probs correct in a table
		3	
(ii)	$E(X) = \frac{-2\times1-1\times2+0+1\times3+2\times2+1\times3}{12} = 0.5$	M1	Unsimplified expression for mean using *their* pdf table (or correct) with at least 2 non-zero values (may be seen in variance). Numerator terms may be implied by values.
	$Var(X) = \frac{(-2)^2\times1+(-1)^2\times2+1^2\times3+2^2\times2+3^2\times1}{12} - (their\ 0.5)^2$	M1	Unsimplified expression for variance using *their* pdf table (or correct) with at least 2 non-zero values and *their* E(X). Numerator terms may be implied by values. If $-k^2$ is seen for $(-k)^2$, the method must be confirmed by seeing value used correctly
	26/12 −1/4 = 23/12	A1	Correct final answer
		3	

Question	Answer	Marks	Guidance
(iii)	**Method 1**		
	P(X non-zero) = 9/12	B1ft	If Binomial distribution used 0/3 P(X non-zero) ft from *their* pdf table, Σp=1 oe
	$P(X=1 \mid X \text{ non-zero}) = \frac{P(X=1 \cap X\ non-zero)}{P(X\ non-zero)} = \frac{3/12}{9/12}$	M1	*Their* P(X = 1)/*their* P(X non-zero) from *their* pdf table oe
	= 1/3 oe	A1	Correct final answer www
	Method 2		
	$P(X=1 \mid X \text{ non-zero}) = \frac{Number\ of\ outcomes = 1}{Number\ of\ non-zero\ outcomes}$	B1ft	Number of non-zero outcomes (expect 9) ft from *their* outcome table or pdf table numerators oe
		M1	a/b, a = *their* 3 from *their* outcome table or pdf table numerators, b = *their* 9 (not 12)
	$= \frac{3}{9} = \frac{1}{3}$ oe	A1	Correct final answer www
		3	

242. 9709_w18_ms_63 Q: 2

Question	Answer	Marks	Guidance
(i)	x: −2, −1, 0, 1, 2, 3 ; P($X=x$): $\frac{2}{18}$, $\frac{4}{18}$, $\frac{5}{18}$, $\frac{4}{18}$, $\frac{2}{18}$, $\frac{1}{18}$	B1	−2, −1, 0, 1, 2, 3 seen as top line of a pdf table OR attempting to evaluate P(−2), P(−1), P(0), P(1), P(2), P(3),
		B1	At least 4 probs correct (need not be in table)
		B1	All probs correct in a table
		3	

Question	Answer	Marks	Guidance
(ii)	$E(X) = \frac{-4-4+0+4+4+3}{18} = \frac{1}{6}$	**M1**	Correct unsimplified expression for the mean using their table, $\Sigma p = 1$, may be implied
	$\text{Var}(X) = \frac{8+4+0+4+8+9}{18} - \left(\frac{1}{6}\right)^2$ $=11/6 - 1/36\ (1.8333 - 0.02778)$	**M1**	Correct, unsimplified expression for the variance using their table, and their mean^2 subtracted. Allow $\Sigma p \neq 1$
	$= 65/36,\ (1.81)$	**A1**	Correct answer
		3	

243. 9709_m17_ms_62 Q: 2

Question	Answer	Marks	Guidance
	$\frac{{}^{12}C_3 \times {}^{28}C_4}{{}^{40}C_7}$	**M1**	Using combinations with attempt to evaluate 2 terms in num. and 1 in denom.
		M1	Correct numerator or denominator unsimplified
	$= 0.242$	**A1**	
	OR		
	$P(GGG) = \frac{12}{40} \times \frac{11}{39} \times \frac{10}{38} \times \frac{28}{37} \times \frac{27}{36} \times \frac{26}{35} \times \frac{25}{34} \times {}^7C_3$	**M1**	Multiplying 3 green probs with 4 non-green probs, without replacement
		M1	Multiplying by ${}^7C_3$
	$= 0.242$	**A1**	
	Total:	**3**	

244. 9709_m17_ms_62 Q: 6

Question	Answer	Marks	Guidance
(i)	$P(2) = P(0,2) = 2/10 \times 4/6$	**M1**	Mult 2 probs seen (or complete listing of all options)
	$= 2/15$ **AG**	**A1**	Correct answer legit obtained
	Total:	**2**	
(ii)	x | 0 | 1 | 2 | 3 | 5 $P(X=x)$ | 2/30 | 5/30 | 4/30 | 13/30 | 6/30	**B1**	Correct values for x in table. Any additional values must have $P(x)=0$ stated
		B1	One correct prob other than P(2) or P(3)
		B1	Correct P(3)
		B1	All correct
	Total:	**4**	
(iii)	$P(A1 \mid \text{Sum } 3) = \frac{P(A1 \cap \text{Sum3})}{P(\text{Sum3})} = \frac{5/10 \times 4/6}{13/30}$	**M1**	Attempt at $P(A1 \cap \text{Sum } 3)$ as num or denom of a fraction, can be by counting
		M1	Their P(3) from **(ii)** as num or denom of a fraction
	$= 10/13 (0.769)$	**A1**	
	Total:	**3**	

245. 9709_s17_ms_61 Q: 5

(i)	$p = 0.07$	**B1**	
	$P(2) = {}^{20}C_2 (0.07)^2 (0.93)^{18}$	**M1**	Bin term ${}^{20}C_x p^x (1-p)^{20-x}$ their p
	$= 0.252$	**A1**	
	Total:	**3**	
(ii)	P(at least 1 cracked egg)=1–$(0.93)^{20}$=1–0.2342	**M1**	Attempt to find P(at least1 cracked egg) with their p from (i) allow 1 – P(0, 1) OE
	$= 0.766$	**A1**	Rounding to 0.766
	Total:	**2**	
(iii)	$(0.7658)^n < 0.01$	**M1**	Eqn or inequal containing (their 0.766)^n or (their 0.234)^n, together with 0.01 or 0.99
	$n = 18$	**A1**	
	Total:	**2**	

246. 9709_s17_ms_62 Q: 3

(i)	$k(-2)^2$ is the same as $k(2)^2 = 4k$	**B1**	need to see $-2^2 k$, $2^2 k$ and $4k$, algebraically correct expressions OE
	Total:	**1**	
(ii)	<table><tr><td>x</td><td>–2</td><td>–1</td><td>2</td><td>4</td></tr><tr><td>Prob</td><td>$4k$</td><td>k</td><td>$4k$</td><td>$16k$</td></tr></table>	**B1**	–2, -1, 2, 4 only seen in a table, together with at least one attempted probability involving k
	$4k + k + 4k + 16k = 1$	**M1**	Summing 4 probs equating to 1. Must all be positive (table not required)
	$k = 1/25$ (0.04)	**A1**	CWO
	Total:	**3**	
(iii)	$E(X) = -8k + -k + 8k + 64k = 63k$	**M1**	using Σpx unsimplified. FT their k substituted before this stage, no inappropriate dividing
	$= 63/25$ (2.52)	**A1**	
	Total:	**2**	

247. 9709_s17_ms_62 Q: 4

Question	Answer	Marks	Guidance
	P(score is 6) = P(3, 3)	**M1**	Realising that score 6 is only P(3, 3)
	$= r^2 = 1/36$ $r = 1/6$	**A1**	Correct ans [SR **B2** $r = 1/6$ without workings]
	P(2, 3) + P(3, 2) = 1/9 $qr + rq = 1/9$	**M1**	Eqn involving qr (OE) equated to 1/9 (r may be replaced by *their* 'r value')
	$q/6 + q/6 = 1/9$	**M1**	Correct equation with *their* 'r value' substituted
	$q = 1/3$	**A1**	Correct answer seen, does **not** imply previous M's
	$p = 1 - 1/6 - 1/3 = 1/2$	**B1 FT**	FT their p + their r + their q =1 , $0 < p < 1$
	Total:	**6**	

248. 9709_s17_ms_62 Q: 7

Question	Answer	Marks	Guidance
(i)	P(H) = P(BH) + P(SH) = 0.6×0.05 + 0.4×0.75	M1	Summing two 2-factor probs using 0.6 with 0.05 or 0.95, and 0.4 with 0.75 or 0.25
	= 0.330 or $\frac{33}{100}$	A1	Correct final answer accept 0.33
	Total:	2	
(ii)	$P(S\|H) = \frac{P(S \cap H)}{P(H)} = \frac{0.4 \times 0.75}{0.33} = \frac{0.3}{0.33}$	M1 FT	Their $\frac{P(S \cap H)}{P(H)}$ unsimplified, FT from (i)
	$= \frac{10}{11}$ or 0.909	A1	
	Total:	2	
(iii)	Var (B) = 45×0.6×0.4 Var (S)= 45×0.4×0.6	B1	One variance stated unsimplified
	Variances same	B1	Second variance stated unsimplified **and** at least one variance clearly identified, **and** both evaluated *or* showing equal *or* conclusion made SR **B1** – Standard Deviation calculated Fulfil all the criteria for the variance method but calculated to Standard Deviation
	Total:	2	

Question	Answer	Marks	Guidance
(iv)	1 – P(0, 1) $= 1 - [(0.6)^{10} + {}^{10}C_1(0.4)(0.6)^9] = 1 - 0.0464$ OR P(2,3,4,5,6,7,8,9,10) $= {}^{10}C_2(0.4)^2(0.6)^8 + \ldots + {}^{10}C_9(0.4)^9(0.6) + (0.4)^{10}$	M1 M1	Bin term ${}^{10}C_x p^x(1-p)^{10-x}$ $0 < p < 1$ Correct unsimplified answer
	= 0.954	A1	
	Total:	3	

249. 9709_s17_ms_63 Q: 5

Question	Answer	Marks	Guidance
(i)	constant probability (of completing)	B1	Any one condition of these two
	independent trials/events	B1	The other condition
	Totals:	2	
(ii)	$P(5, 6, 7) = {}^7C_5(0.7)^5(0.3)^2 + {}^7C_6(0.7)^6(0.3)^1 + (0.7)^7$	M1 A1	Bin term ${}^7C_x(0.7)^x(0.3)^{7-x}$, $x \neq 0, 7$ Correct unsimplified answer (sum) OE
	= 0.647	A1	
	Total:	3	
(iii)	P(0, 1, 2, 3, 4) = 1 – their '0.6471' = 0.3529	M1	Find P($\leqslant 4$) either by subtracting their (ii) from 1 or from adding Probs of 0,1,2,3,4 with n=7 (or 10) and p = 0.7
	$P(3) = {}^{10}C_3(0.3529)^3(0.6471)^7$	M1	${}^{10}C_3$ (their 0.353)^3(1 – their 0.353)^7 on its own
	= 0.251	A1	

250. 9709_s17_ms_63 Q: 6

(a)(i)	First digit in 2 ways. $2 \times 4 \times 3 \times 2$ or $2 \times 4P3$	**M1**	1, 2 or $3 \times 4P3$ OE as final answer
	Total = 48 ways	**A1**	
	Total:	**2**	
(a)(ii)	$2 \times 5 \times 5 \times 3$	**M1** **M1**	Seeing 5^2 mult; this mark is for correctly considering the middle two digits with replacement Mult by 6; this mark is for correctly considering the first and last digits
	= 150 ways	**A1**	
	Totals:	**3**	

Question	Answer	Marks	Guidance
(b)(i)	OO**** in $^{18}C_4$ ways	**M1**	$^{18}C_x$ or the sum of five 2-factor products with $n = 14$ and 4, may be × by 2C2: 4C0 × 14C4 + 4C1 × 14C3 + 4C2 × 14C2 + 4C3 × 14C1 + 4C4 (× 14C0)
	= 3060	**A1**	
	Totals:	**2**	

Question	Answer	Marks	Guidance
(b)(ii)	Choc / Not Choc 0 / $6 = 1 \times {}^{16}C_6 = 8008$ 0.2066 1 / $5 = {}^{4}C_1 \times {}^{16}C_5 = 17472$ 0.4508 2 / $4 = {}^{4}C_2 \times {}^{16}C_4 = 10920$ 0.2817 OR Choc / Oats / Ginger 0 / 0 / 6 0 / 1 / 5 0 / 2 / 4 1 / 0 / 5 1 / 1 / 4 1 / 2 / 3 2 / 0 / 4 2 / 1 / 3 2 / 2 / 2	**B1**	The correct number of ways with one of 0, 1 or 2 chocs , unsimplified **or** any three correct number of ways of combining choc/oat/ginger, unsimplified
	Total = 36400 ways	**M1**	sum the number of ways with 0, 1 and 2 chocs and two must be totally correct, unsimplified OR sum the nine combinations of choc, ginger, oats, six must be totally correct, unsimplified
	Probability = $36400 / {}^{20}C_6$	**M1**	dividing by $^{20}C_6$ (38760) oe
	= 0.939 (910/969)	**A1**	
	Totals:	**4**	

251. 9709_w17_ms_61 Q: 1

Question	Answer	Marks	Guidance
	$p + q = 0.45$	**M1**	Equation involving $\Sigma P(x) = 1$
	$0.15 + 2p + 1.2 + 6q = 3.05$	**M1**	Equation using $E(X) = 3.05$
	$q = 0.2$	**M1**	Solving simultaneous equations to one variable
	$p = 0.25$	**A1**	Both answers correct
		4	

252. 9709_w17_ms_61 Q: 3

Question	Answer	Marks	Guidance
(i)	$p = 0.207$	B1	
		1	
(ii)	Var = $30 \times 0.207 \times 0.793 = 4.92$	B1	
		1	
(iii)	$P(\geqslant 2) = 1 - P(0, 1)$	M1	
	$= 1 - (0.793)15 - \binom{15}{1}(0.207)(0.793)14$	M1	$1 - P(0, 1)$ seen $n = 15$ p = any prob
	$= 0.848$	A1	
		3	

253. 9709_w17_ms_62 Q: 3

Question	Answer	Marks	Guidance
(i)	*EITHER:* $P(X = 3) = P(RRB) = \frac{2}{6} \times \frac{1}{5} \times \frac{4}{4}$	(M1	probabilities in order $\frac{2}{p} \times \frac{1}{q} \times \frac{4}{r}$, $p, q, r \leqslant 6$ and $p \geqslant q \geqslant r$, $r \geqslant 4$, accept $\times 1$ as $\frac{4}{r}$.
	$= \frac{1}{15}$ AG	A1)	Needs either P(RRB) OE stated or identified on tree diagram.
	OR1: $P(X = 3) = P(RRB) = \frac{{}^2C_2}{{}^6C_2} \times \frac{{}^4C_1}{{}^4C_1}$	(M1	probabilities stated clearly, $\times \frac{{}^4C_1}{{}^4C_1}$ or $\times 1$ or $\times \frac{4}{4}$ included
	$= \frac{1}{15}$ AG	A1)	Needs either P(RRB) OE stated or identified on tree diagram.
	OR2: $P(X = 3) = P(RRB) = \frac{{}^2C_1}{{}^6C_1} \times \frac{{}^1C_1}{{}^5C_1} \times \frac{{}^4C_1}{{}^4C_1}$	(M1	probabilities in order $\frac{{}^2C_1}{{}^pC_1} \times \frac{{}^1C_1}{{}^qC_1} \times \frac{{}^4C_1}{{}^rC_1}$ $p, q, r \leqslant 6$ and $p \geqslant q \geqslant r$, $r \geqslant 4$ ($\times \frac{{}^4C_1}{{}^4C_1}$ or $\times 1$ or $\times \frac{4}{4}$ acceptable)
	= 1/15 AG	A1)	Needs either P(RRB) OE stated or identified on tree diagram.
		2	

Question	Answer	Marks	Guidance
(ii)	$P(1) = P(B) = \frac{4}{6}$ $(\frac{2}{3} = 0.667)$	B1	Probability distribution table drawn with at least 2 correct x values and at least 1 probability. All probabilities $0 \leqslant p < 1$.
	$P(2) = P(RB) = \frac{2}{6} \times \frac{4}{5} = \frac{4}{15}$ $(= 0.267)$	B1	P(1) or P(2) correct unsimplified, or better, and identified.
	$P(3) = P(RRB) = \frac{2}{6} \times \frac{1}{5} \times \frac{4}{4} = \frac{1}{15}$ $(= 0.0667)$	B1	All probabilities in table, evaluated correctly OE. Additional x values must have a stated probability of 0
	x | 1 | 2 | 3 P | $\frac{10}{15}$ | $\frac{4}{15}$ | $\frac{1}{15}$		
		3	

254. 9709_w17_ms_62 Q: 4

Question	Answer	Marks	Guidance
(i)	$P(4, 2H) = \frac{1}{4} \times {}^4C_2 \times (\frac{1}{3})^2(\frac{2}{3})^2$	M1	Multiplying their 2H expression by ¼ [P(4)]
		M1	Remaining factor is $(\frac{1}{3})^2(\frac{2}{3})^2$ [or $\frac{4}{81}$] multiplied by integer value $k \geqslant 1$ OE
	$= \frac{2}{27}$ (0.0741)	A1	
		3	
(ii)	$P(3, 3H) = \frac{1}{4} \times (\frac{1}{3})^3 = \frac{1}{108}$ (0.00926)	B1	
		1	
(iii)	$P(1, 1H) = \frac{1}{4} \times \frac{1}{3} = \frac{1}{12}$ (0.08333) $P(2, 2H) = \frac{1}{4} \times (\frac{1}{3})^2 = \frac{1}{36}$ (0.02778) $P(3, 3H) = \frac{1}{4} \times (\frac{1}{3})^3 = \frac{1}{108}$ (0.009259) $P(4, 4H) = \frac{1}{4} \times (\frac{1}{3})^4 = \frac{1}{324}$ (0.003086)	M1	Correct expression for 1 of P(1, 1H), P(2, 2H), P(4, 4H) Unsimplified (or better)
		M1	Summing their values for 3 or 4 appropriate outcomes for the 'game' with no additional outcomes.
	Prob = $\frac{10}{81}$ (0.123)	A1	
		3	

255. 9709_w17_ms_63 Q: 1

Question	Answer	Marks	Guidance
	EITHER: P(at least 1 completes) = 1 – P(0 people complete) $= 1 - (0.8)^3$	(M1	Fully correct unsimplified expression $1 - (0.8)^3$ OE
	$= 0.488 \left(\frac{61}{125}\right)$	A1)	
	OR1: $P(1, 2, 3) = {}^3C_1(0.2)(0.8)^2 + {}^3C_2(0.2)^2(0.8) + (0.2)^3$	(M1	Unsimplified correct 3 term expression
	$= 0.488 \left(\frac{61}{125}\right)$	A1)	
	OR2: $0.2 + 0.8 \times 0.2 + 0.8 \times 0.8 \times 0.2$	(M1	Unsimplified sum of 3 correct terms
	$= 0.488 \left(\frac{61}{125}\right)$	A1)	
		2	

256. 9709_w17_ms_63 Q: 4

Question	Answer	Marks	Guidance
(i)	x | –3 | 0 | 5 | 32 Prob | 1/6 | 1/2 | 1/6 | 1/6	B1	At least 3 different correct values of X (can be unsimplified)
		B1	Four correct probabilities in a Probability Distribution table
		B1	Correct probs with correct values of X
		3	

Question	Answer	Marks	Guidance
(ii)	$E(X) = -3/6 + 5/6 + 32/6 = 34/6 = 17/3$ (5.67)	**M1**	Subst their attempts at scores in correct formula as long as 'probs' sum to 1
	$Var(X) = 9/6 + 25/6 + 1024/6 - (34/6)^2$	**M1**	Subst their attempts at scores in correct var formula
	$= 144\left(\frac{1298}{9}\right)$	**A1**	Both answers correct
		3	

257. 9709_m16_ms_62 Q: 2

	No of W: 0, 1, 2 Prob: 42/90, 42/90, 6/90	**B1**	0, 1, 2, seen in table with attempt at prob.
	$P(0) = 8/10 \times 7/9 \times 6/8 = 42/90$	**M1**	3-factor prob seen with different denoms.
	$P(1W) = P(W,NW, NW) \times 3 = 2/10 \times 8/9 \times 7/8 \times 3$ $= 42/90$	**M1**	Mult by 3
	$P(2W) = P(W, W, NW) \times 3 = 2/10 \times 1/9 \times 8/8 \times 3$ $= 6/90$	**A1** 4	All correct

258. 9709_s16_ms_61 Q: 2

	P (throwing a 4) = (1 – 0.4) / 4 = 0.15	**M1** **A1**	Sensible attempt to find P(1) Correct answer
	P(at most 1) = P(0, 1) or 1 – P(2, 3) $= (0.85)^3 + {}^3C_1\ (0.15)\ (0.85)^2$	**M1** **M1**	A binomial term with ${}^3C_n$ oe any p Binomial expression with ${}^3C_n$ P(0, 1) or 1 – P(2, 3) $p = 0.15$ or 0.85
	= 0.939	**A1** [5]	

259. 9709_s16_ms_61 Q: 4

	$[P(X = 0)] = P(B, B) = 5/7 \times 4/6 = 10/21$	**M1**	Attempt to find P(0) or P(1) or P(2) can be seen as P(BB) etc. or table unsimplified
	$[P(X = 1)] = P(G,B) + P(B,G) = 2/7 \times 5/6 \times 2 = 10/21$	**A1**	P(1) or P(BG)+P(GB) correct
	$[P(X = 2] = P(G, G) = 2/7 \times 1/6 = 1/21$	**A1**	P(0) or P(2) correct must see X value
	$E(X) = 0 + 10/21 + 2/21 = 4/7$ (0.571)	**B1**√	Correct answer ft their probs P(1) and P(2)
	$Var(X) = 0 + 10/21 + 4/21 - (4/7)^2$ = 50/147 (0.340)	**M1** **A1** [6]	Attempt at $\Sigma x^2 p - [E(X)]^2$

260. 9709_s16_ms_62 Q: 3

(i)	x | 1 | 2 | 3 | 4 P(x) | k | $2k$ | $3k$ | $4k$ $10k = 1$ $k = 1/10$	**B1** **M1** **A1** [3]	Probability Distribution Table, either k or correct numerical values Summing probs involving k to = 1, 3 or 4 terms	
(ii)	$E(X) = 1/10 + 4/10 + 9/10 + 16/10 = 3$ $Var(X) = 1/10 + 8/10 + 27/10 + 64/10 - 3^2$ $= 1$	**B1** **M1** **A1** [3]	Correct mean Correct method seen for var, their k and μ	

261. 9709_s16_ms_62 Q: 4

(i)	$p = 0.66$ $X \sim$ B(15, 0.66) P(at least 14) = P(14, 15) = $^{15}C_{14}\,(0.66)^{14}(0.34) + (0.66)^{15}$ $= 0.0171$	**M1** **M1** **A1** [3]	Bin term $^{15}C_x p^x (1-p)^{15-x}$ seen any p Unsimplified correct expression for P(14,15)
(ii)	$(0.87)^n < 0.04$ $n = 24$	**M1** **M1** **A1** [3]	Eqn involving 0.87, power of n, 0.04 only Solving by logs or trial and error(can be implied). Must be exponential equation

262. 9709_s16_ms_63 Q: 3

(i)	P(0) = 6/36, P(1) = 10/36, P(2) = 8/36 P(3) = 6/36, P(4) = 4/36, P(5) = 2/36	**B1** **B1** **M1** **A1** [4]	Table oe seen with 0, 1, 2, 3, 4, 5 (6 if P(6) = 0) Any three probs correct $\Sigma p = 1$ and at least 3 outcomes All probs correct
(ii)	mean score = (0×6+1×10 +16 +18 +16+10)/36 = 70/36 (35/18, 1.94)	**M1** **A1** [2]	Using Σxp (unsimplified) on its own – condone Σp not =1

263. 9709_w16_ms_61 Q: 1

	$z = 0.674$ $0.674 = \frac{k-20}{7}$ $k = 24.7$	**M1** **M1** **A1**	[3]	±0.674 seen Standardising no cc, no sq, no sq rt

264. 9709_w16_ms_61 Q: 2

	diff: 0, 1, 2, 3, 4, 5 prob: 6/36, 10/36, 8/36, 6/36, 4/36, 2/36 Expectation = (0+10+16+18+16+10)/36 = 70/36 = 1.94	B1 M1 A1 M1 A1	[5]	0, 1, 2, 3, 4, 5 seen in table heading or considering all different differences Attempt at finding prob of any difference 1 correct prob Probs summing to 1

265. 9709_w16_ms_61 Q: 3

(i)	$0.9 \times 0.95 \times 0.85 \times 0.1 = 0.0727$	B1	[1]	
(ii)	P(0, 1, 2) $= (0.9)^{12} + {}^{12}C_1\,(0.1)(0.9)^{11} + {}^{12}C_2\,(0.1)^2(0.9)^{10}$ $= 0.889$	M1 M1 A1	[3]	Bin term ${}^{12}C_x\,(p)^x(1-p)^{12-x}$ $p < 1$, $x \neq 0$ Bin expression $p = 0.1$ or 0.9, $n = 12$, 2 or 3 terms
(iii)	$X \sim B(50, 0.85)$ Expectation $= 50 \times 0.85$ (= 42.5) Var $= 50 \times 0.85 \times 0.15$ (= 6.375)	M1 A1	[2]	50×0.85 seen oe can be implied Correct unsimplified mean and var

266. 9709_w16_ms_62 Q: 2

(i)	P(1 T-shirt) $= \dfrac{{}^3C_1 \times {}^9C_2}{{}^{12}C_3}$ $= 27/55$ AG **OR** $3/12 \times 9/11 \times 8/10 \times {}^3C_1$ oe $= 27/55$ AG	B1 B1 B1 M1 M1 A1	[3]	Correct num unsimplified Correct denom unsimplified Answer given, so process needs to be convincing Mult 3 probs diff denoms (not a/3 x b/4 x c/5) Mult by ${}^3C_1$ oe Answer given, so process needs to be convincing
(ii)	X: 0, 1, 2, 3 Prob: 84/220, 27/55, 27/220, 1/220	B1 B1 B1 B1√	[4]	0, 1, 2, 3 only seen in top line (condone additional values if Prob stated as 0) One correct prob, correctly placed in table One other correct prob, correctly placed in table One other correct prob ft $\Sigma p = 1$, 4 values in table

267. 9709_w16_ms_63 Q: 2

(i)	$p = 1/3$ $P(\geqslant 2) = 1 - P(0, 1) = 1 - (2/3)^4 - {}^4C_1(1/3)(2/3)^3$ or $P(2,3,4) = {}^4C_2(1/3)^2(2/3)^2 + {}^4C_3(1/3)^3(2/3) + (1/3)^4$ $= \dfrac{11}{27}$, 0.407	M1 M1 A1	[3]	Bin term ${}^4C_x p^x(1-p)^{4-x}$ $0 < p < 1$ Correct unsimplified answer
(ii)	$P(\text{sum is } 5) = P(1, 1, 1, 2) \times 4 = (1/3)^4 \times 4$ $= \dfrac{4}{81}$, 0.0494	M1 M1 A1	[3]	1, 1, 1, 2 seen or 4 options Mult by $(1/3)^4$

268. 9709_s15_ms_62 Q: 1

	P(3, 4, 5) =	M1	Bin expression of form ${}^{10}C_x (p)^x (1-p)^{10-x}$ any x any p
	${}^{10}C_3\left(\frac{1}{6}\right)^3\left(\frac{5}{6}\right)^7 + {}^{10}C_4\left(\frac{1}{6}\right)^4\left(\frac{5}{6}\right)^6 + {}^{10}C_5\left(\frac{1}{6}\right)^5\left(\frac{5}{6}\right)^5$	A1	Correct unsimplified answer accept (0.17, 0.83), (0.16, 0.84), (0.16, 0.83), (0.17, 0.84) or more accurate
	= 0.222	A1 **3**	Correct answer

269. 9709_s15_ms_62 Q: 5

(i)	$P(2Es\ 1O) = \frac{3}{5}\times\frac{2}{4}\times\frac{2}{3}\times{}^3C_2 = \frac{3}{5}$ (0.6)	M1 M1 A1 **3**	5×4×3 seen in denom Mult a prob by ${}^3C_2$ oe Correct answer
	OR $P(2Es\ 1O) = \frac{{}^3C_2\times{}^2C_1}{{}^5C_3} = \frac{6}{10}$	M1	${}^3C_x$ or ${}^yC_2$ or ${}^2C_1$ oe seen mult by $k \geqslant 1$ in num
	= 0.6	M1 A1	${}^5C_3$ seen in denom Correct answer
	OR 241, 247, 261, 267, 461, 467 = 6 options 124 126 127 146 147 167 246 247 267 467	M1 M1	List at least 3 of 241, 247, 261, 267, 461, 467 ${}^5C_3$ or list to get all 10 options in denom see below
	Prob = 6/10	A1	Correct answer
(ii)	124 126 127 146 147 167 246 247 267 467 s | 1 | 2 | 4 $P(S=s)$ | 6/10 | 3/10 | 1/10	M1 A1 B1 B1 B1 **5**	Attempt at listing with at least 7 correct All correct and no others or all 60 1, 2, 4 only seen in top row Any two correct All correct

270. 9709_s15_ms_63 Q: 2

(i)			All values may be decimals or %
	Kitchen mess | Kitchen not mess | Total On time | 1/10 | 1/10 | Not on time | 1/2 | | 4/5 Total | 3/5 | 4/10 |	B1	2 probabilities correct
		B1	2 further probabilities correct
		B1 **[3]**	2 further probabilities correct
(ii)	P(not on time given kitchen mess) = $\frac{1/2}{3/5}$	M1	A cond prob fraction seen (using corresponding combined outcomes and total)
	= 5/6 o.e.	A1 **[2]**	FT from their values, 3sf or better, <1, 3/5ft<1

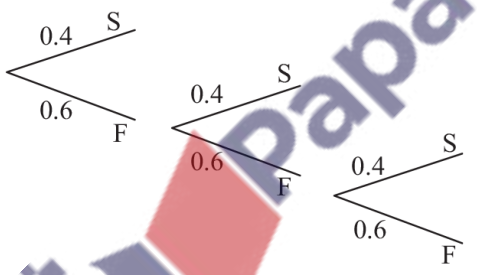
271. 9709_s15_ms_63 Q: 4

(i)	P(1 W) = 6/9×3/8 + 3/9×6/8	M1	summing 2 two-factor probs (condone replacement) not ½×½ + ½×½
	= ½ AG	A1 **[2]**	Correct answer, fully justified
	OR $\frac{{}^6C_1 \times {}^3C_1}{{}^9C_2}$	M1	Using combinations consistent, correct format
	= ½ AG	A1	Correct answer, fully justified
(ii)	$P(\overline{W},\overline{W})$ = 3/9×2/8 = 6/72 (1/12) $P(W,W)$ = 6/9×5/8 = 30/72 (5/12) x | 0 | 1 | 2 Prob | 1/12 | 1/2 | 5/12	B1 B1 B1 √ **[3]**	Distribution table with 0,1,2 only $P(W,W)$ or $P(\overline{W},\overline{W})$ correct $P(W,W) + P(\overline{W},\overline{W}) = 0.5$
(iii)	E(X) = 16/12 (4/3) (1.33) isw	B1 **[1]**	Condone 1(.3) if correct working seen, nfww

272. 9709_w15_ms_61 Q: 1

	$p = 0.76$ P(fewer than 10) = 1 – P(10, 11) $= 1 - (0.76)^{10}(0.24)^{11}C_{10} - (0.76)^{11}$ = 1 – 0.219 = 0.781	**M1** **M1** **M1** **A1** [4]	Any binomial term ${}^{11}C_x p^x(1\text{-}p)^{11-x}$, $0 < p < 1$ Any binomial term ${}^nC_x(0.76)^x(0.24)^{n-x}$ 1 – P(10, 11) oe binomial expression Correct answer

273. 9709_w15_ms_61 Q: 6

(i)		**M1** **A1** **A1** [3]	3 pairs S (bank, log in, success oe) and F oe seen no extra bits. Exactly 3 pairs, must be labelled Correct diagram with all probs correct
(ii)	x | 0 | 1 | 2 | 3 Prob | 0.4 | | 0.144 | 0.216	**B1** **M1** **A1** **B1** [4]	P(0) correct Multiplying two of more factors of 0.4 and 0.6 One more correct prob One more correct prob
(iii)	E(X) = 0.24+ 2× 0.144 + 3× 0.216 = 1.176 (1.18)	**M1** **A1** [2]	Using $\Sigma p_i x_i$ Correct answer

274. 9709_w15_ms_62 Q: 3

(i)	$\frac{1}{4}$	**B1**	1	
(ii)	$\left(\frac{3}{4}\right)^4\left(\frac{1}{4}\right) = \frac{81}{1024} = 0.0791$	**M1** **A1**	 2	Expression of form $p^4(1-p)$ only, $p = 1/4$ or $3/4$ Correct answer
(iii)	P(all diff) $= \frac{1}{4}\times\frac{1}{4}\times\frac{1}{4}\times\frac{1}{4}\times 4!$ $= \frac{3}{32}$ (0.0938) OR $1\times\frac{3}{4}\times\frac{2}{4}\times\frac{1}{4} = \frac{3}{32}$	**M1** **M1** **A1**	 3	4! on numerator seen mult by k ⩾ 1 or 3×2×1 on num oe, must be in a fraction. 4^4 on denom or 4^3 on denom with the 3× 2× 1 Correct answer

275. 9709_w15_ms_62 Q: 6

(i)	Spinner *A* Spinner *B* –3: (–2) –1 0 0 –2: –1 0 (1) 1 –1: 0 1 2 2 1: 2 3 4 4	**B1**	1	
(ii)	x: –2 –1 0 1 2 3 3 4 prob: $\frac{1}{16}$ $\frac{2}{16}$ $\frac{4}{16}$ $\frac{3}{16}$ $\frac{3}{16}$ $\frac{1}{16}$ $\frac{2}{16}$	**M1** **M1** **A1**	 3	Their values in (i) as the top line, seen listed in (ii) or used in part (iii) Attempt at probs seen evaluated, need at least 4 correct from their table Correct table seen
(iii)	E(X) = 1 Var(X) $= ((-2)^2 + 2 + 3 + 12 + 9 + 32)/16 - 1^2$ $= \frac{62}{16} - 1$ $= \left(\frac{23}{8}\right)$ (2.875) OR using $\Sigma p(x-\bar{x})^2 = (9 + 8 + 4 + 0 + 3 + 4 + 18)/16$ $= \frac{46}{16} = 2.875$	**M1** **M1** **A1** **M1** **M1** **A1**	 	Attempt at E(X) from their table if $\Sigma p = 1$ Evaluating $\Sigma x^2 p$ – [their E(X)]^2 allow $\Sigma p \neq 1$ but all p's <1 Correct answer

(iv)	P(even given +ve) $= \frac{5}{9}$	**M1**		Counting their even numbers and dividing by their positive numbers
		A1	2	Correct answer
	OR P(even given +ve) $= \frac{\left(\frac{5}{16}\right)}{\left(\frac{9}{16}\right)}$	**M1**		Using cond prob formula not P(E) × P(+ve) need fraction over fraction accept any of $\frac{5/16 or 6/16 or 9/16}{9/16 or 10/16 or 13/16}$
	$= \frac{5}{9}(0.556)$	**A1**		Correct answer

276. 9709_m22_ms_52 Q: 4

Question	Answer	Marks	Guidance
(a)	$P(46 < X < 62) = P\left(\frac{46-55}{6} < Z < \frac{62-55}{6}\right)$	**M1**	46 or 62, 55 and 6 substituted into ±standardisation formula once. Condone 6^2 and continuity correction ±0.5
	$= P\left(-1.5 < Z < \frac{7}{6}\right)$	**B1**	Both standardisation values correct, accept unsimplified
	$\left[=\Phi\left(\frac{7}{6}\right)-\left(1-\Phi(1.5)\right)\right]$ $= 0.8784+(0.9332-1)$	**M1**	Calculating the appropriate area from stated Φs of z-values, must be probabilities.
	0.812	**A1**	$0{\cdot}8115 < p \leqslant 0{\cdot}812$
		4	
(b)	$z = \pm 0.674$	**B1**	CAO, critical z-value
	$\frac{36-42}{\sigma} = -0.674$	**M1**	36 and 42 substituted in ±standardisation formula, no continuity correction, not σ^2, $\sqrt{\sigma}$, equated to a z-value
	$\sigma = 8.9[0]$	**A1**	WWW. Only dependent on M.
		3	

Question	Answer	Marks	Guidance
(c)	P(male < 46) = 1−*their* 0.9332 = 0.0668	**M1**	FT value from part **(a)** or Correct: $1-\Phi\left(\frac{46-55}{6}\right)$,condone continuity correction, σ^2, $\sqrt{\sigma}$, **and** probability found. Condone unsupported correct value stated.
	P(female < 46) = P$\left(Z < \frac{46-42}{\textit{their}\ 8.90}\right)\left[=\Phi(0.449)\right]$ $= 0.6732$	**M1**	46, 42 and *their* 4(b) σ (or correct σ) substituted in ±standardisation formula, condone continuity correction, σ^2, $\sqrt{\sigma}$, **and** probability found Condone $\frac{4}{\textit{their}\ 8.90}$.
	P(both) = 0.0668 ×0.6732	**M1**	Product of *their* 2 probabilities (0 < both < 1) Not 0.25 or *their* final answer to **4(a)** used.
	0.0450 or 0.0449	**A1**	$0{\cdot}0449 \leqslant p \leqslant 0{\cdot}0450$
		4	

277. 9709_s22_ms_51 Q: 5

Question	Answer	Marks	Guidance
(a)	$P(X<6)=P(Z<\frac{6-5.2}{1.5})=P(Z<0.5333)$	M1	6, 5.2, 1.5 substituted into ± standardisation formula, condone 1.5^2, continuity correction ±0.5
	0.703	A1	
		2	
(b)	$z_1=\frac{3-\mu}{\sigma}=-1.329$ $z_2=\frac{8-\mu}{\sigma}=0.878$	B1	$1.328<z_1\leqslant 1.329$ or $-1.329\leqslant z_1<-1.328$
		B1	$0.877<z_2\leqslant 0.878$ or $-0.878\leqslant z_2<-0.877$
	Solve to find at least one unknown: $\frac{3-\mu}{\sigma}=-1.329$ $\frac{8-\mu}{\sigma}=0.878$	M1	Use of the ± standardisation formula once with μ, σ, a z-value (not 0.8179, 0.7910, 0.5367, 0.5753, 0.19, 0.092 etc.) and 3 or 8, condone continuity correction but not σ^2 or $\sqrt{\sigma}$
		M1	Use either the elimination method or the substitution method to solve their two equations in μ and σ
	$\sigma=2.27,\quad \mu=6.01$	A1	$2.26\leqslant\sigma\leqslant 2.27$, $6.01\leqslant\mu\leqslant 6.02$
		5	

Question	Answer	Marks	Guidance
(c)	$[P(Z<-1)+P(Z>1)]\ \Phi(1)-\Phi(-1)=$ $=2-2\ \Phi(1)$ $=2-2\times 0.8413$	M1	Identify 1 and –1 as the appropriate z-values.
		M1	Calculating the appropriate area from stated phis of z-values which must be ± the same number
	0.3174	A1	Accept AWRT 0.317
	Number of leaves: $2000\times 0.3174=634.8$ so 634 or 635	B1 FT	FT *their* 4 s.f. (or better) probability, final answer must be positive integer no approximation or rounding stated
		4	

278. 9709_s22_ms_52 Q: 4

Question	Answer	Marks	Guidance
(a)	$[P(1.98<X<2.03)=]P(\frac{1.98-2.02}{0.03}<z<\frac{2.03-2.02}{0.03})$ $[=P(-1.333<z<0.333)]$	M1	Use of ±standardisation formula once with 2.02, 0.03 and either 1.98 or 2.03 substituted appropriately. Condone 0.03^2 and continuity correction ±0.005, not $\sqrt{0.03}$.
	$[=\Phi(0.333)-(1-\Phi(1.333))]$ $=0.6304+0.9087-1$	M1	Calculating the appropriate probability area from *their* z-values. (or 0.6304 – 0.09121 or (0.9087 – 0.5) + (0.6304 – 0.5) etc)
	0.539	A1	$0.539\leqslant z<0.5395$ Only dependent upon 2nd M mark. If M0 scored **SC B1** for $0.539\leqslant z<0.5395$.
		3	
(b)	$[P(X>2.6)=\frac{134}{5000}=0.0268]$ $[P(X<2.6)=1-0.0268=]\ 0.9732$	B1	0.9732 or $\frac{4866}{5000}$ or $\frac{2433}{2500}$ seen.
	$\frac{2.6-2.55}{\sigma}=1.93$	M1	Use of ±standardisation formula with 2.6 and 2.55 substituted, no $\sigma^2,\sqrt{\sigma}$ or continuity correction.
		M1	*Their* standardisation formula with values substituted equated to z-value which rounds to ±1.93.
	$\sigma=0.0259$	A1	AWRT 0.0259 or $\frac{5}{193}$. If M0 earned, **SC B1** for correct final answer.
		4	

279. 9709_s22_ms_52 Q: 5

Question	Answer	Marks	Guidance
(a)	[P(10, 11, 12) =] $^{12}C_{10}\,0.72^{10}\,0.28^{2} + {}^{12}C_{11}\,0.72^{11}\,0.28^{1} + {}^{12}C_{12}\,0.72^{12}\,0.28^{0}$	M1	One term ${}^{12}C_x\,p^x\,(1-p)^{12-x}$, for $0<x<12$, $0<p<1$.
	$= 0.193725 + 0.0905726 + 0.0194084$	A1	Correct expression, accept unsimplified, no terms omitted, leading to final answer.
	0.304	B1	Final answer $0.3036 < p \leqslant 0.304$.
	Alternative method for question 5(a)		
	[1 – P(0,1,2,3,4,5,6,7,8,9) =] $1 - ({}^{12}C_0\,0.72^{0}\,0.28^{12} + {}^{12}C_1\,0.72^{1}\,0.28^{11} + {}^{12}C_2\,0.72^{2}\,0.28^{10} + {}^{12}C_3\,0.72^{3}\,0.28^{9} + {}^{12}C_4\,0.72^{4}\,0.28^{8} + {}^{12}C_5\,0.72^{5}\,0.28^{7} + {}^{12}C_6\,0.72^{6}\,0.28^{6} + {}^{12}C_7\,0.72^{7}\,0.28^{5} + {}^{12}C_8\,0.72^{8}\,0.28^{4} + {}^{12}C_9\,0.72^{9}\,0.28^{3})$	M1	One term ${}^{12}C_x\,p^x\,(1-p)^{12-x}$, for $0<x<12$, $0<p<1$.
		A1	Correct expression, accept unsimplified, no terms omitted, leading to final answer.
	0.304	B1	Final answer $0.3036 < p \leqslant 0.304$.
		3	
(b)	Mean = [0.52×90 =] 46.8, var = [$0.52\times0.48\times90$] = 22.464	B1	46.8 and 22.464 or 22.46 seen, allow unsimplified, ($4.739 < \sigma \leqslant 4.740$ imply correct variance).
	$[P(X<40) =]\ P\left(z < \frac{39.5-46.8}{\sqrt{22.464}}\right)$	M1	Substituting *their* mean and *their* variance into ±standardisation formula (any number for 39.5), not σ^2, $\sqrt{\sigma}$.
		M1	Using continuity correction 39.5 or 40.5 in *their* standardisation formula.
	$= [P(Z<-1.540)] = 1 - 0.9382$	M1	Appropriate area Φ, from final process, must be probability.
	0.0618	A1	$0.06175 \leqslant p \leqslant 0.0618$
		5	

280. 9709_s22_ms_53 Q: 5

Question	Answer	Marks	Guidance
(a)	$[P(142<X<205)] = P\left(\frac{142-170}{25} < z < \frac{205-170}{25}\right)$	M1	Use of ± standardisation formula once substituting 170, 25 and either 142 or 205 appropriately.. Condone 25^2 and continuity correction ±0.5.
	$P(-1.12 < z < 1.4)$	A1	Both correct. Accept unsimplified.
	$\Phi(1.4) - (1 - \Phi(1.12)) = 0.9192 + 0.8686 - 1$	M1	Calculating the appropriate area from stated phis of *z*-values.
	0.788	A1	AWRT, not from wrong working
		4	
(b)	$P(X > 205) = 1 - 0.9192 = 0.0808$	B1 FT	Correct or FT from part **5(a).**
	$(0.0808\times0.30 + \textit{their}\ 0.788\times0.24)\times20000$	M1	Correct or *their* 0.0808 × 0.30 × *k* + *their* 0.788 × 0.24 × *k*, *k* positive integer.
	[$]4266.24	A1	4265 < income ⩽ 4270, not from wrong working
		3	
(c)	$[P(Z > \frac{w-182}{20}) = 0.72]$ $\frac{w-182}{20} = -0.583$	B1	$0.5828 \leqslant z \leqslant 0.583$ or $-0.583 \leqslant z \leqslant -0.5828$ seen.
		M1	182 and 20 substituted in ± standardisation formula, no continuity correction, not σ^2, $\sqrt{\sigma}$, equated to a *z*-value.
	$w = 170$	A1	$170 \leqslant w < 170.35$
		3	

281. 9709_m21_ms_52 Q: 3

Question	Answer	Marks	Guidance
(a)	$P\left(\left(\frac{85-96}{18}\right)<z<\left(\frac{100-96}{18}\right)\right)$	**M1**	Use of ±standardisation formula once with appropriate values substituted, no continuity correction, not σ^2 or $\sqrt{\sigma}$.
	$P(-0.6111<z<0.2222)$ $=\Phi(0.2222)+\Phi(0.6111)-1$ $=0.5879+0.7294-1$	**M1**	Appropriate area Φ, from final process, must be probability. Use of $(1-z)$ implies M0.
	0·317	**A1**	Final answer which rounds to 0·317.
		3	

Question	Answer	Marks	Guidance
(b)	$z=\pm1{\cdot}175$	**B1**	$1{\cdot}17 \leqslant z \leqslant 1{\cdot}18$ or $-1{\cdot}18 \leqslant z \leqslant -1{\cdot}17$
	$-1.175=\frac{t-96}{18}$	**M1**	An equation using ±standardisation formula with a z-value, condone σ^2, $\sqrt{\sigma}$ or continuity correction. E.g. equating to 0·88, 0·12, 0·8106, 0·1894, 0·5478, 0·4522, ±0·175 or ±2·175 implies M0.
	74·85 or 74·9	**A1**	$74{\cdot}85 \leqslant t \leqslant 74{\cdot}9$
		3	

282. 9709_m21_ms_52 Q: 7

Question	Answer	Marks	Guidance
(a)(i)	$\left[\frac{104+31}{400}=\right]\frac{135}{400}, \frac{27}{80}, 0{\cdot}3375$	**B1**	Evaluated, exact value.
		1	
(a)(ii)	**Method 1**		
	$P(M)=\frac{180}{400}, 0{\cdot}45$ $P(S)=\frac{135}{400}, 0{\cdot}3375$ $P(M\cap S)=\frac{31}{400}, 0{\cdot}0775$ $\frac{180}{400}\times\frac{135}{400}=\frac{243}{1600}, 0.151875\neq\frac{31}{400}$ so NOT independent	**M1**	*Their* P(M) × *their* P(S) seen, accept unsimplified.
		A1	P(M), P(S) and P($M \cap S$) notation seen, numerical comparison and correct conclusion, WWW.
	Method 2		
	$P(M\cap S)=\frac{31}{400}$ $P(S)=\frac{135}{400}$ $P(M)=\frac{180}{400}$ $P(M\|S)=\frac{\frac{31}{400}}{\frac{135}{400}}=\frac{31}{135}, 0.2296\ldots\neq\frac{180}{400}$ so NOT independent	**M1**	$[P(M\|S)=]\frac{\text{their } P(M\cap S)}{\text{their } P(S)}$ (oe) seen, accept unsimplified.
		A1	P(M), P(S) and P($M \cap S$) notation seen, numerical comparison and correct conclusion, WWW.
		2	

Question	Answer	Marks	Guidance
(b)(i)	**Method 1** [1 – P(0,1,2)]		
	$= 1 - ({}^{10}C_0\,0{\cdot}3^0\,0{\cdot}7^{10} + {}^{10}C_1\,0{\cdot}3^1\,0{\cdot}7^9 + {}^{10}C_2\,0{\cdot}3^2\,0{\cdot}7^8)$	**M1**	${}^{10}C_x\,p^x\,(1-p)^{10-x}$ for $0 < x < 10$, $0 < p < 1$, any p.
	$= 1 - (0{\cdot}028248 + 0{\cdot}121061 + 0{\cdot}233474)$	**A1**	Correct expression, accept unsimplified, condone omission of final bracket, condone recovery from poor notation.
	$= 0{\cdot}617$	**A1**	Accept $0{\cdot}61715 \leqslant p \leqslant 0{\cdot}61722$, WWW.
	Method 2 [P(3,4,5,6,7,8,9,10) =]		
	${}^{10}C_3\,0{\cdot}3^3\,0{\cdot}7^7 + {}^{10}C_4\,0{\cdot}3^4\,0{\cdot}7^6 + {}^{10}C_5\,0{\cdot}3^5\,0{\cdot}7^5 + {}^{10}C_6\,0{\cdot}3^6\,0{\cdot}7^4 + {}^{10}C_7\,0{\cdot}3^7\,0{\cdot}7^3 + {}^{10}C_8\,0{\cdot}3^8\,0{\cdot}7^2 + {}^{10}C_9\,0{\cdot}3^9\,0{\cdot}7^1 + {}^{10}C_{10}\,0{\cdot}3^{10}\,0{\cdot}7^0$	**M1**	${}^{10}C_x\,p^x\,(1-p)^{10-x}$ for $0 < x < 10$, $0 < p < 1$, any p.
		A1	Correct unsimplified expression.
	$= 0{\cdot}617$	**A1**	Accept $0{\cdot}61715 \leqslant p \leqslant 0{\cdot}61722$, WWW.
		3	

Question	Answer	Marks	Guidance
(b)(ii)	[p = 0·3] Mean = 0·3 × 90 = 27; variance = 0·3 × 90 × 0·7 = 18·9	**B1**	Correct mean and variance, allow unsimplified. Condone $\sigma = 4{\cdot}347$ evaluated.
	$P(X < 32) = P\left(z < \frac{31.5-27}{\sqrt{18.9}}\right)$	**M1**	Substituting *their* μ and σ (not σ^2, $\sqrt{\sigma}$) into the ±standardising formula with a numerical value for '31·5'.
		M1	Using either 31·5 or 32·5 within a ±standardising formula with numerical values for *their* μ and σ (condone σ^2, $\sqrt{\sigma}$).
	$= \Phi(1.035)$	**M1**	Appropriate area Φ, from standardisation formula P(*z*<…) in final solution, must be probability.
	$= 0{\cdot}850$	**A1**	Allow $0{\cdot}8495 < p \leqslant 0{\cdot}85(0)$, final answer WWW.
		5	

283. 9709_s21_ms_51 Q: 2

Question	Answer	Marks	Guidance
	$\left[P\left(\left(\frac{25.2-(25.5+0.50)}{0.4}\right) < z < \left(\frac{25.2-(25.2-0.50)}{0.4}\right)\right)\right]$ $= P\left(-\frac{0.5}{0.4} < z < \frac{0.5}{0.4}\right)$	M1	Use of ± Standardisation formula once; no continuity correction, σ^2, $\sqrt{\sigma}$
	$[= 2\Phi(1.25) - 1]$ $= 2 \times 0.8944 - 1$	A1	For AWRT 0.8944 SOI
		M1	Appropriate area 2Φ – 1 OE, from final process, must be probability
	0.7888	A1	Accept AWRT 0.789
	Number of rods = 0.7888×500 = 394 or 395	B1FT	Correct or FT *their* 4SF (or better) probability, final answer must be positive integer, not 394.0 or 395.0, no approximation/rounding stated, only 1 answer
		5	

284. 9709_s21_ms_51 Q: 6

Question	Answer	Marks	Guidance
(a)	$1 - P(10, 11, 12) = 1 - ({}^{12}C_{10}\,0.6^{10}\,0.4^{2} + {}^{12}C_{11}\,0.6^{11}\,0.4^{1} + {}^{12}C_{12}\,0.6^{12}\,0.4^{0})$ [= 1 – (0.063852 + 0.017414 + 0.0021768)]	M1	One term: ${}^{12}C_x\,p^x\,(1-p)^{12-x}$ for $0 < x < 12$, any p allowed.
		A1	Correct unsimplified expression, or better.
	[1 – 0.083443] = 0.917	A1	AWRT
	Alternative method for Question 6(a)		
	P (0,1,2,3,4,5,6,7,8,9) = ${}^{12}C_0 0.6^0\,0.4^{12} + {}^{12}C_1\,0.6^1\,0.4^{11} + \ldots\ldots\ldots\ldots {}^{12}C_9\,0.6^9\,0.4^3$ [= 0.000016777 + 0.00030199 + 0.0024914 + 0.012457 + 0.042043 + 0.10090 + 0.17658 + 0.22703 + 0.21284 + 0.14189]	M1	One term: ${}^{12}C_x\,p^x\,(1-p)^{12-x}$ for $0 < x < 12$, any p allowed.
		A1	Correct unsimplified expression with at least the first two and last terms
	0.917	A1	WWW, AWRT
		3	

Question	Answer	Marks	Guidance
(b)	[Mean =] 0.6 × 150 [= 90]; [Variance =] 0.6 × 150 × 0.4 [= 36]	B1	Correct mean and variance. Accept evaluated or unsimplified
	$P(X<81) = P\left(Z < \frac{80.5-90}{6}\right)$	M1	Substituting *their* mean and variance into ±standardisation formula (with a numerical value for 80.5), allow σ^2, $\sqrt{\sigma}$, but not $\mu \pm 0.5$
		M1	Using continuity correction 80.5 or 81.5
	$\Phi(-1.5833) = 1 - 0.9433$	M1	Appropriate area Φ, from final process, must be probability
	0.0567	A1	AWRT
		5	
(c)	$np = 90$, $nq = 60$ both greater than 5	B1	At least nq evaluated and statement >5 required
		1	

285. 9709_s21_ms_52 Q: 2

Question	Answer	Marks	Guidance
	$\left[P(X>1.1) = \frac{72}{2000} (= 0.036)\right]$ $z = \pm 1.798$	B1	$1.79 < z \leqslant 1.80$, $-1.80 \leqslant z < -1.79$ seen
	$\frac{1.1-1.04}{\sigma} = 1.798$ $\left[\frac{0.06}{\sigma} = 1.798\right]$	B1	1.1 and 1.04 substituted in ±standardisation formula, allow continuity correction, not σ^2 or $\sqrt{\sigma}$
		M1	Equate *their* ±standardisation formula to a z-value and to solve for the appropriate area leading to final answer (expect $\sigma < 0.5$). $\left(\text{Accept} \pm \frac{0.06}{\sigma} = z - \text{value}\right)$
	$\sigma = 0.0334$	A1	$0.03335 \leq \sigma \leq 0.0334$. At least 3 3s.f.
		4	

286. 9709_s21_ms_52 Q: 5

Question	Answer	Marks	Guidance
(a)	$[(0.7)^3 =]0.343$	B1	Evaluated WWW
	Alternative method for Question 5(a)		
	$[(0.15)^3 + {}^3C_1(0.15)^2(0.55) + {}^3C_2(0.15)(0.55)^2 + (0.55)^3 =] 0.343$	B1	Evaluated WWW
		1	
(b)	$1 - (0.85^9 + {}^9C_1\,0.15^1\,0.85^8 + {}^9C_2\,0.15^2\,0.85^7)$ $[1 - (0.231617 + 0.367862 + 0.259667)]$	M1	One term: ${}^9C_x\,p^x\,(1-p)^{9-x}$ for $0 < x < 9$, any $0 < p < 1$
		A1	Correct expression, accept unsimplified.
	0.141	A1	$0.1408 \leqslant$ ans $\leqslant 0.141$, award at most accurate value.
	Alternative method for Question 5(b)		
	${}^9C_3\,0.15^3\,0.85^6 + {}^9C_4\,0.15^4\,0.85^5 + {}^9C_5\,0.15^5\,0.85^4 + {}^9C_6\,0.15^6\,0.85^3 +$ ${}^9C_7\,0.15^7\,0.85^2 + {}^9C_8\,0.15^8\,0.85 + 0.15^9$	M1	One term: ${}^9C_x\,p^x\,(1-p)^{9-x}$ for $0 < x < 9$, any $0 < p < 1$
		A1	Correct expression, accept unsimplified.
	0.141	A1	$0.1408 \leqslant$ ans $\leqslant 0.141$, award at most accurate value.
		3	

Question	Answer	Marks	Guidance
(c)	Mean $= [60 \times 0.15 =] 9$ Variance $= [60 \times 0.15 \times 0.85 =] 7.65$	B1	Correct mean and variance, allow unsimplified. ($2.765 \leq \sigma \leq 2.77$ imply correct variance)
	$[(X \geq 12) =] P\left(Z > \frac{11.5-9}{\sqrt{7.65}}\right)$	M1	Substituting *their* mean and variance into ±standardisation formula (any number for 11.5), not σ^2 or $\sqrt{\sigma}$
		M1	Using continuity correction 11.5 or 12.5 in *their* standardisation formula.
	$1 - \Phi(0.9039) = 1 - 0.8169$	M1	Appropriate area Φ, from final process, must be probability.
	0.183	A1	Final AWRT
		5	

287. 9709_s21_ms_53 Q: 5

Question	Answer	Marks	Guidance
(a)	$z_1 = \frac{4-\mu}{\delta} = -1.378$	B1	$1.378 \leqslant z_1 \leq 1.379$ or $-1.379 \leqslant z_1 \leqslant -1.378$
	$z_2 = \frac{10-\mu}{\sigma} = 0.842$	B1	$0.841 \leqslant z_2 \leqslant 0.842$ or $-0.842 \leqslant z_2 \leqslant -0.841$
	Solve to find at least one unknown: $\frac{4-\mu}{\sigma} = -1.378$ $\frac{10-\mu}{\sigma} = 0.842$	M1	Use of ±standardisation formula once with μ, σ, a z-value and 4 or 10, allow continuity correction, not σ^2 or $\sqrt{\sigma}$
		M1	Use either the elimination method or the substitution method to solve two equations in μ and σ.
	$\sigma = 2.70 \quad \mu = 7.72$	A1	$2.70 \leqslant \sigma \leqslant 2.71 \quad 7.72 \leqslant \mu \leqslant 7.73$
		5	
(b)	$\Phi(2) - \Phi(-2) = 2\Phi(2) - 1$	M1	Identifying 2 and –2 as the appropriate z-values
	$2 \times \textit{their}\ 0.9772 - 1$	B1	Calculating the appropriate area from stated phis of z-values which must be ± the same number
	0.9544 or 0.9545	A1	Accept AWRT 0.954
	$0.9544 \times 800 = 763.52$ 763 or 764	B1 FT	FT *their* 4SF (or better) probability, final answer must be positive integer
		4	

288. 9709_s21_ms_53 Q: 7

Question	Answer	Marks	Guidance
(a)(i)	$\frac{40}{800}$ or $\frac{1}{20}$ or 0.05	B1	
		1	
(a)(ii)	$\frac{177}{223+177+40}$	M1	*Their* 223 + 177 + 40 seen as denominator of fraction in the final answer, accept unsimplified
	$\frac{177}{440}$ or 0.402	A1	CAO
	Alternative method for Question 7(a)(ii)		
	$P(\mathrm{G} \mid \mathrm{S}) = \frac{P(G \cap S)}{P(S)} = \frac{\frac{177}{800}}{\frac{223+177+40}{800}} = \frac{\frac{177}{800}}{\frac{440}{800}} = \frac{\frac{177}{800}}{\frac{11}{20}}$ or 0.55	M1	*Their* P(S) seen as denominator of fraction in the final answer, accept unsimplified
	$\frac{177}{440}$ or 0.402	A1	CAO
		2	
7(b)(i)	P(0, 1, 2) = $^{10}C_0 (0.35)^0 (0.65)^{10} + {}^{10}C_1 (0.35)^1 (0.65)^9 + {}^{10}C_2 (0.35)^2 (0.65)^8$	M1	One term: $^{10}C_x p^x (1-p)^{10-x}$ for $0 < x < 10$, any $0<p<1$
	0.013463 + 0.072492 + 0.17565	A1	Correct unsimplified expression, or better
	0.262	A1	
		3	

Question	Answer	Marks	Guidance
(b)(ii)	Mean = $120\times0.35[=42]$ Variance = $120\times0.35\times0.65[=27.3]$	**B1**	Correct mean and variance seen, allow unsimplified
	$P(X>32) = P(Z > \frac{32.5-42}{\sqrt{27.3}}) = P(Z > -1.818)$	**M1**	Substituting *their* mean and variance into ±standardisation formula (any number), condone σ^2 or $\sqrt{\sigma}$
		M1	Using continuity correction 31.5 or 32.5
	$\Phi(1.818)$	**M1**	Appropriate area Φ, from final process, must be probability
	0.966	**A1**	$0.965 \leqslant p \leqslant 0.966$
		5	

289. 9709_w21_ms_51 Q: 7

Question	Answer	Marks	Guidance
(a)(i)	$P(X>142) = P\left(Z > \frac{142-125}{24}\right)$	**M1**	Substitution of correct values into the ±Standardisation formula, allow continuity correction, not σ^2, $\sqrt{\sigma}$.
	$[= P(Z > 0.7083) =]1-0.7604$	**M1**	Appropriate numerical area Φ, from final process, must be probability, expect $p < 0.5$
	0.2396	**A1**	$0.239 \leqslant p \leqslant 0.240$ to at least 3sf.
	Their 0.2396×365 [= 87.454]	**M1**	**FT** *their* 4sf (or better) probability.
	87 or 88	**A1 FT**	Final answer must be positive integer, no indication of approximation/rounding, only dependent on previous **M** mark. **SC B1 FT** for *their* 3sf probability × 365 = integer value, condone 0.24 used.
		5	
(a)(ii)	$P(0, 1) = 0.7604^{10} + {}^{10}C_1 \times 0.2396^1 \times 0.7604^9$ [= 0.064628 + 0.20364]	**M1**	One term: ${}^{10}C_x p^x (1-p)^{10-x}$ for $0 < x < 10$, any p.
		A1 FT	Correct unsimplified expression using *their* probability to at least 3sf from **(a)(i)** or correct.
	0.268	**A1**	AWRT, WWW.
		3	
(b)	$z = \pm1.282$	**B1**	Correct value only, critical value.
	$\frac{t-125}{24} = -1.282$	**M1**	Use of ± Standardisation formula with correct values substituted, allow continuity correction, σ^2, $\sqrt{\sigma}$, to form an equation with a z-value and not probability.
	$t = 94.2$	**A1**	AWRT, condone AWRT 94.3. Not dependent on **B** mark.
		3	

290. 9709_w21_ms_52 Q: 6

Question	Answer	Marks	Guidance
(a)	$[P(X > 28.6) =] P\left(Z > \frac{28.6 - 32.2}{9.6}\right)$ $[= P(Z > -0.375)]$	**M1**	28.6, 32.2 and 9.6 substituted appropriately in ± Standardisation formula once, allow continuity correction of ± 0.05, no σ^2, $\sqrt{\sigma}$.
	[Φ(*their* 0.375) =] *their* 0.6462	**M1**	Appropriate numerical area, from final process, must be probability, expect > 0.5.
	0.646	**A1**	AWRT
		3	
(b)	$z = \pm 0.842$	**B1**	0.841 < z ⩽ 0.842 or -0.842 ⩽ z < –0.841 seen.
	$\frac{t - 32.2}{9.6} = 0.842$	**M1**	Substituting 32.2 and 9.6 into ± standardisation formula, no continuity correction, allow σ^2, $\sqrt{\sigma}$, must be equated to a *z*-value.
	$t = 40.3$	**A1**	40.28 ⩽ *t* ⩽ 40.3 WWW
		3	

Question	Answer	Marks	Guidance
(c)	$P\left(-\frac{15}{9.6} < Z < \frac{15}{9.6}\right)$ $P(-1.5625 < Z < 1.5625)$	**M1**	Identifying at least one of $\frac{15}{9.6}$ and $-\frac{15}{9.6}$ as the appropriate z-values or substituting *their* (32.2 ± 15) into ± Standardisation formula once, no continuity correction, σ^2 nor $\sqrt{\sigma}$. Condone ±1.563 for **M1**.
	$[2\ \Phi(\frac{15}{9.6}) - 1\ ]$ $= 2 \times 0.9409 - 1$	**A1**	$p = 0.941$ AWRT SOI
		M1	Appropriate area $2\Phi - 1$ oe, (eg $1 - 2 \times 0.0591$, $2 \times (0.9409 - 0.5)$ or $0.9409 - 0.0591$), from final process, must be probability > 0.5.
	0.882	**A1**	
		4	

291. 9709_w21_ms_53 Q: 4

Question	Answer	Marks	Guidance
(a)	$P(X > 43.2) = P\left(Z > \frac{43.2 - 41.2}{3.6}\right) = P(Z > 0.5556)$	**M1**	Use of ±Standardisation formula once, allow continuity correction, not σ^2, $\sqrt{\sigma}$.
	$1 - \Phi(0.5556) = 1 - 0.7108$	**M1**	Appropriate area Φ, from final process, must be probability.
	0.289	**A1**	AWRT
		3	
(b)	Probability = 1 – *their* **(a)** = 1 – 0.2892 = 0.7108	**B1FT**	1 – *their* **(a)** or correct.
	$0.7108 \times 365 = 259.4$ 259, 260	**B1FT**	FT *their* 4SF (or better) probability, final answer must be positive integer.
		2	
(c)	$z = \pm 1.645$	**B1**	CAO, critical *z* value.
	$\frac{t - 41.2}{3.6} = -1.645$	**M1**	Use of ±standardisation formula with μ, σ equated to a *z*-value, no continuity correction, allow σ^2, $\sqrt{\sigma}$.
	$t = 35.3$	**A1**	
		3	

292. 9709_m20_ms_52 Q: 3

Question	Answer	Marks	Guidance
(a)	$P(X > 87) = P\left(Z > \frac{87-82}{\sigma}\right) = 0.22$	M1	Using ± standardisation formula, not σ^2, not $\sqrt{\sigma}$, no continuity correction
	$P\left(Z < \frac{5}{\sigma}\right) = 0.78$ $\left(\frac{5}{\sigma} =\right) 0.772$	B1	AWRT ±0.772 seen B0 for ±0.228
	$\sigma = 6.48$	A1	
		3	
(b)	$P\left(-\frac{4}{\sigma} < Z < \frac{4}{\sigma}\right) = P(-0.6176 < Z < 0.6176)$	M1	Using ±4 used within a standardisation formula (SOI), allow σ^2, $\sqrt{\sigma}$ and continuity correction
		M1	Standardisation formula applied to **both** *their* ±4
	$\Phi = 0.7317$ Prob $= 2\Phi - 1 = 2(0.7317) - 1$	M1	Correct area $2\Phi - 1$ oe linked to final solution
	$= 0.463$	A1	
		4	

293. 9709_m20_ms_52 Q: 5

Question	Answer	Marks	Guidance
(a)	$1 - P(6, 7, 8)$ $= 1 - ({}^8C_6\, 0.7^6 0.3^2 + {}^8C_7\, 0.7^7 0.3^1 + 0.7^8)$	M1	One term ${}^8C_x\, p^x (1-p)^{8-x}$, $0 < p < 1$, $x \neq 0$
	$= 1 - 0.55177$	A1	Correct unsimplified expression, or better
	$= 0.448$	A1	
	Alternative method for question 5(a)		
	$P(0, 1, 2, 3, 4, 5)$ $= 0.3^8 + {}^8C_1\, 0.7^1 0.3^7 + {}^8C_2\, 0.7^2 0.3^6 + {}^8C_3\, 0.7^3 0.3^5 + {}^8C_4 0.7^4 0.3^4 + {}^8C_5\, 0.7^5 0.3^3$	M1	One term ${}^8C_x\, p^x (1-p)^{8-x}$, $0 < p < 1$, $x \neq 0$
		A1	Correct unsimplified expression, or better
	$= 0.448$	A1	
		3	
(b)	Mean $= 120 \times 0.7 = 84$ Var $= 120 \times 0.7 \times 0.3 = 25.2$	B1	Correct mean and variance, allow unsimplified
	$P(\text{more than } 75) = P\left(z > \frac{75.5 - 84}{\sqrt{25.2}}\right)$	M1	Substituting *their* μ and σ into the ±standardising formula (any number), not σ^2, not $\sqrt{\sigma}$
		M1	Using continuity correction 75.5 or 74.5
	$P(z > -1.693)$	M1	Appropriate area Φ, from final process, must be a probability
	$= 0.955$	A1	Allow $0.9545 < p \leqslant 0.955$
		5	

294. 9709_s20_ms_51 Q: 6

Question	Answer	Marks
(a)	$P\left(\frac{50-54}{6.1} < z < \frac{60-54}{6.1}\right) = P(-0.6557 < Z < 0.9836)$	M1
	Both values correct	A1
	$\Phi(0.9836) - \Phi(-0.6557) = \Phi(0.9836) + \Phi(0.6557) - 1$ $= 0.8375 + 0.7441 - 1$ (Correct area)	M1
	0.582	A1
		4

Question	Answer	Marks
(b)	$\frac{45-\mu}{\sigma} = -0.994$	B1
	$\frac{56-\mu}{\sigma} = 1.372$	B1
	One appropriate standardisation equation with μ, σ, z-value (not probability) and 45 or 56.	M1
	$11 = 2.366\,\sigma$ (**M1** for correct algebraic elimination of μ or σ from *their* two simultaneous equations to form an equation in one variable)	M1
	$\sigma = 4.65, \mu = 49.6$	A1
		5

295. 9709_s20_ms_52 Q: 4

Question	Answer	Marks
(a)	$P(X < 25) = P\left(z < \frac{25-40}{12}\right) = P(z < -1.25) P(X < 25) = P(z <)$	M1
	$1 - 0.8944$	M1
	0.106	A1
		3
(b)	0.8944 divided by 3 (**M1** for 1 - *their* **(a)** divided by 3)	M1
	0.298 **AG**	A1
		2
(c)	0.2981 gives $z = 0.53$	B1
	$\frac{h-40}{12} = 0.53$	M1
	$h = 46.4$	A1
		3

296. 9709_s20_ms_52 Q: 7

Question	Answer	Marks
(a)	$1 - P(10, 11, 12)$ $= 1 - [{}^{12}C_{10}\,0.72^{10}0.28^{2} + {}^{12}C_{11}\,0.72^{11}0.28^{1} + 0.72^{12}]$	M1
	$1 - (0.19372 + 0.09057 + 0.01941)$	A1
	0.696	A1
		3
(b)	$0.28^{3} \times 0.72 = 0.0158$	B1
		1

Question	Answer	Marks
(c)	Mean $= 100 \times 0.72 = 72$ Var $= 100 \times 0.72 \times 0.28 = 20.16$	M1
	P(less than 64) $= P\left(z < \frac{63.5 - 72}{\sqrt{20.16}}\right)$ (**M1** for substituting *their* μ *and* σ into ±standardisation formula with a numerical value for '*63.5*')	M1
	Using either 63.5 or 64.5 within a ±standardisation formula	M1
	Appropriate area Φ, from standardisation formula P(z<…) in final solution $= P(z < -1.893)$	M1
	0.0292	A1
		5

297. 9709_s20_ms_53 Q: 3

Question	Answer	Marks
(a)	$P(X < 21) = P\left(z < \frac{21 - 15.8}{4.2}\right) = \Phi(1.238)$	M1
	0.892	A1
		2
(b)	$z = \pm 0.674$	B1
	$\frac{k - 15.8}{4.2} = 0.674$	M1
	18.6	A1
		3

298. 9709_s20_ms_53 Q: 5

Question	Answer	Marks
(a)	$\frac{1}{\frac{1}{4}} = 4$	B1
		1
(b)	$\frac{9}{64}$ $(=0.141)$	B1
		1
(c)	$P(X < 6) = 1 - \left(\frac{3}{4}\right)^5$ (**FT** *their* probability/mean from part **(a)**)	M1
	0.763	A1
		2
(d)	Mean $= 80 \times 0.25 = 20$ Var $= 80 \times 0.25 \times 0.75 = 15$	M1
	P(more than 25) $= P\left(z > \frac{25.5 - 20}{\sqrt{15}}\right)$	M1
	$P(z > 1.42)$	M1
	$1 - 0.9222$	M1
	0.0778	A1
		5

299. 9709_w20_ms_51 Q: 5

(a)	$P(X > 4.2) = P\left(z > \frac{4.2 - 3.5}{0.9}\right)$ $= P(z > 0.7778)$	M1	Using ± standardisation formula, no $\sqrt{\sigma}$ or σ^2, continuity correction
	$1 - 0{\cdot}7818$	M1	Appropriate area Φ, from standardisation formula P(*z*>…) in final solution
	0·218	A1	
		3	
(b)	$z = -1.282$	B1	±1.282 seen (critical value)
	$\frac{t - 3.5}{0.9} = -1.282$	M1	An equation using ±standardisation formula with a *z*-value, condone $\sqrt{\sigma}$, σ^2 and continuity correction
	$t = 2.35$	A1	AWRT, only dependent on M mark
		3	

Question	Answer	Marks	Guidance
(c)	P(2.8 < X < 4.2) = 1 − 2 × *their* **5(a)** ≡ 2(1 − *their* **5(a)**) − 1 ≡ 2(0·5 − *their* **5(a)**) = 0·5636	**B1 FT**	FT from *their* **5(a)** < 0.5 or correct Accept unevaluated probability OE Accept 0·564
	Number of days = 365 × 0·5636 = 205·7	**M1**	365 × *their p*
	So, 205 (days)	**A1 FT**	Accept 205 or 206, not 205·0 or 206·0 no approximation/ rounding stated FT must be an integer value
	Alternative method for question 5(c)		
	$P\left(\frac{2.8-3.5}{0.9}<z<\frac{4.2-3.5}{0.9}\right)$ $=\Phi(0.7778)-(1-\Phi 0.7778)$ = 0·7818 − (1 − 0·7818) = 0·5636	**B1**	0·5635 < *p* ⩽ 0·564 OE
	Number of days = 365 × 0·5636 = 205·7	**M1**	365 × *their p*
	So, 205 (days)	**A1 FT**	Accept 205 or 206, not 205·0 or 206·0 no approximation/ rounding stated FT must be an integer value
		3	

300. 9709_w20_ms_52 Q: 3

Question	Answer	Marks	Guidance
(a)	$P(X > 11.3) = P(z > \frac{11.3-10.1}{1.3}) = P(z > 0\cdot 9231)$	**M1**	Using ± standardisation formula, no $\sqrt{\sigma}$ or σ^2, continuity correction
	1 − 0.822	**M1**	Appropriate area Φ, from standardisation formula P(*z*>…) in final solution
	0·178	**A1**	0.1779…
		3	
(b)	$z = -0\cdot 674$	**B1**	±0.674 seen (critical value)
	$\frac{t-10.1}{1.3} = -0\cdot 674$	**M1**	An equation using ±standardisation formula with a *z*-value, condone $\sqrt{\sigma}$ or σ^2, continuity correction.
	t = 9·22	**A1**	AWRT. Only dependent on M1
		3	

Question	Answer	Marks	Guidance
(c)	P(8.9 < X < 11.3) = 1 – 2 × *their* **3(a)** ≡ 2(1 – *their* **3(a)**) – 1 ≡ 2(0·5 – *their* **3(a)**) =0.644	**B1 FT**	FT from *their* **3(a)** < 0·5 or correct, accept unevaluated probability OE
	Number of days = 90 × 0·644 = 57·96	**M1**	90 × *their p* seen, 0 < p < 1
	So 57 (days)	**A1 FT**	Accept 57 or 58, not 57·0 or 58·0, no approximation/rounding stated FT must be an integer value
	Alternative method for question 3(c)		
	$P\left(\frac{8\cdot9-10\cdot1}{1\cdot3}<z<\frac{11.3-10.1}{1\cdot3}\right)$ $=\Phi(0\cdot9231)-(1-\Phi(0\cdot9231))$ oe = 0·822 – (1 – 0·822) = 0·644	**B1**	Accept unevaluated probability
	Number of days = 90 × 0·644 = 57·96	**M1**	90 × *their p* seen, 0 < p < 1
	So 57 (days)	**A1 FT**	Accept 57 or 58, not 57·0 or 58·0, no approximation/rounding stated FT must be an integer value
		3	

301. 9709_w20_ms_53 Q: 1

Question	Answer	Marks	Guidance
(a)	$P(56<X<66)=P\left(\frac{56-62}{5}<z<\frac{66-62}{5}\right)$ $=P(-1.2<z<0.8)$	**M1**	Using ± standardisation formula at least once, no $\sqrt{\sigma}$ or σ^2, allow continuity correction
	$\Phi(0.8)+\Phi(1.2)-1$ = 0.7881 + 0.8849 – 1	**M1**	Appropriate area Φ, from standardisation formula in final solution
	0.673	**A1**	
		3	
(b)	$z=1.127$	**B1**	±(1.126 – 1.127) seen, 4 sf or more
	$\frac{60t-62}{5}=1.127$ $60t$ = 5.635+62=67.635	**M1**	z-value = $\pm\frac{(60t-62)}{5}$ condone z-value = $\pm\frac{(t-62)}{5}$ no continuity correction, condone $\sqrt{\sigma}$ or σ^2
	t = 1.13	**A1**	CAO
		3	

302. 9709_w20_ms_53 Q: 4

Question	Answer	Marks	Guidance
(a)	$0.65^7 + {}^7C_1\ 0.65^6\ 0.35^1 + {}^7C_2\ 0.65^5\ 0.35^2$	M1	Binomial term of form ${}^7C_x\ p^x(1-p)^{7-x}$, $0<p<1$, any p, $x \neq 0, 7$
	0.049022 + 0.184776 + 0.29848	A1	Correct unsimplified answer
	0.532	A1	
		3	
(b)	Mean = $142\times0.35 = 49.7$ Variance = $142\times0.35\times0.65 = 32.305$	B1	Correct unsimplified *np* and *npq* (condone $\sigma = 5.684$ evaluated)
	$P(X>40) = P(z > \frac{40.5-49.7}{\sqrt{32.305}})$	M1	Substituting *their* μ *and* σ (no $\sqrt{\sigma}$ or σ^2) into ±standardisation formula with a numerical value for '40.5'
	$P(z>-1.619)$	M1	Using either 40.5 or 39.5 within a ±standardisation formula
		M1	Appropriate area Φ, from standardisation formula $P(z>\ldots)$ in final solution, must be probability
	0.947	A1	Correct final answer
		5	

303. 9709_m19_ms_62 Q: 3

Question	Answer	Marks	Guidance
(i)	$P(X<132) = P\left(Z < \frac{132-140}{12}\right) = P(Z<-0.6667)$	M1	Using ± standardisation formula, no continuity correction, not σ^2 or $\sqrt{\sigma}$
	= 1 – 0.7477	M1	Appropriate area Φ from standardisation formula P(z<….) in final solution
	= 0.252 awrt	A1	Condone linear interpolation = 0.25243
		3	
(ii)	P(time>k) = 0.675, $z = -0.454$	B1	±0.454 seen
	$\frac{k-140}{12} = -0.454$	M1	An equation using the standardisation formula with a z-value (not $1-z$), condone σ^2 or $\sqrt{\sigma}$
	$k = 135, 134.6, 134.55$	A1	B0M1A1 max from –0.45
		3	

304. 9709_m19_ms_62 Q: 6

Question	Answer	Marks	Guidance
(i)	$P(4, 5, 6) = {}^6C_4\,0.35^4 0.65^2 + {}^6C_5\,0.35^5 0.65^1 + 0.35^6$	M1	Binomial term of form ${}^6C_x p^x(1-p)^{6-x}$ $0 < p < 1$ any p, $x \neq 6,0$
		A1	Correct unsimplified answer
	= 0.117	A1	
		3	
(ii)	$1 - 0.65^n > 0.95$ $0.65^n < 0.05$	M1	Equation or inequality involving '0.65^n or 0.35^n' and '0.95 or 0.05'
	$n > \dfrac{\log 0.05}{\log 0.65} = 6.95$	M1	Attempt to solve *their* exponential equation using logs or Trial and Error.
	$n = 7$	A1	CAO
		3	
(iii)	Mean = $0.35 \times 100 = 35$ Variance = $0.35 \times 0.65 \times 100 = 22.75$	B1	Correct unsimplified *np* and *npq*,
	$P\left(z > \dfrac{39.5 - 35}{\sqrt{22.75}}\right) = P(z > 0.943)$	M1	Substituting *their* μ and σ (condone σ^2) into the ±Standardisation Formula with a numerical value for *'39.5'*.
		M1	Using continuity correction 39.5 or 40.5
	$= 1 - 0.8272$	M1	Appropriate area Φ from standardisation formula P(z>….) in final solution, (>0.5 if z is -ve, <0.5 if z is +ve)
	= 0.173	A1	Final answer
		5	

305. 9709_s19_ms_61 Q: 5

Question	Answer	Marks	Guidance
(i)	$(P > 12) = P(13, 14, 15)$	M1	Binomial term of form ${}^{15}C_x p^x(1-p)^{15-x}$ $0 < p < 1$ any p, $x \neq 15,0$
	$= {}^{15}C_{13}(0.65)^{13}(0.35)^2 + {}^{15}C_{14}(0.65)^{14}(0.35)^1 + (0.65)^{15}$	A1	Correct unsimplified answer
	= 0.0617	A1	SC if use np and npq with justification give (12.5 – 9.75)/√3.41 M1 1–F(1.489) A1 0.0681 A0
		3	
(ii)	mean = 250 × 0.65 = 162.5 variance = 250 × 0.65 × 0.35 = 56.875	B1	Correct unsimplified *np* and *npq*
	$P(< 179) = P\left(z < \dfrac{178.5 - 162.5}{\sqrt{56.875}}\right) = P(z < 2.122)$	M1	Substituting *their* μ and σ (condone σ^2) into the Standardisation Formula with a numerical value for *'178.5'*. Continuity correct not required for this M1. Condone ± standardisation formula
	Using continuity correction 178.5 or 179.5	M1	
	= 0.983	A1	Correct final answer
		4	

306. 9709_s19_ms_61 Q: 7

Question	Answer	Marks	Guidance
(i)	$P(< 700) = P\left(z < \dfrac{700 - 830}{120}\right) = P(z < -1.083)$	M1	Using ± standardisation formula, no continuity correction, not σ^2 or $\sqrt{\sigma}$
	= 1 – 0.8606	M1	Appropriate area Φ from standardisation formula P(z<….) in final probability solution, (<0.5 if z is –ve, >0.5 if z is +ve)
	= 0.1394	A1	Correct final probability rounding to 0.139
	Expected number of female adults = 430 × *their* 0.1394 = 59.9 So 59 or 60	B1	**FT** *their* 3 or 4 SF probability, rounded or truncated to integer
		4	

Question	Answer	Marks	Guidance
(ii)	P(giraffe < 830+w) = 95% so z = 1.645	**B1**	±1.645 seen (critical value)
	$\frac{(830+w)-830}{120} = \frac{w}{120} = 1.645$	**M1**	An equation using the standardisation formula with a z-value (not 1 – z), condone σ^2 or $\sqrt{\sigma}$ not 0.8519, 0.8289
	w = 197	**A1**	Correct answer
		3	
(iii)	P(male > 950) = 0.834, so z = –0.97	**B1**	± 0.97 seen
	$\frac{950-1190}{\sigma} = -0.97$	**M1**	Using ± standardisation formula, condone continuity correction, σ^2 or $\sqrt{\sigma}$, condone equating with non z-value not 0.834, 0.166
	σ = 247	**A1**	Condone $-\sigma$ = –247. www.
		3	

307. 9709_s19_ms_62 Q: 2

Question	Answer	Marks	Guidance
	$P(<28.9) = P\left(z < \frac{28.9-30}{1.5}\right)$	**B1**	Using ± standardising formula, no continuity correction, not σ^2 or $\sqrt{\sigma}$,
	= P(z < –0.733) = 1 – 0.7682	**M1**	Appropriate area Φ from standardisation formula P(z <….) in final probability solution, Must be a probability, e.g. 1 – 0.622 is M0
	= 0.2318	**A1**	Correct final probability rounding to 0.232. (Only requires M1 not B1 to be awarded
	Number of cartridges is *their* 0.2318 × 8 = 1.85, so 2 (Also accept 1 but not both)	**B1**	**FT** using *their* 4 SF (or better) value, ans. rounded or truncated to integer, no approximation indicated.
		4	

308. 9709_s19_ms_62 Q: 4

Question	Answer	Marks	Guidance
	$z = 0.842 = \left(\frac{121-\mu}{\sigma}\right)$ so $0.842\sigma = 121 - \mu$	**B1**	± 0.842 seen but B0 if 1 ± 0.842 oe seen
		M1	One appropriate standardisation equation with a z-value, μ, σ and 121 **or** 102, condone continuity correction. Not 0.158, 0.42,…
	$z = -0.58 = \left(\frac{102-\mu}{\sigma}\right)$ so $-0.58\sigma = 102 - \mu$	**B1**	± 0.58(0) seen but B0 if 1 ± 0.58 oe seen
	Solving	**M1**	Correct algebraic elimination of μ or σ from *their* two simultaneous equations to form an equation in one variable, condone 1 numerical slip
	σ = 13.4 μ = 110	**A1**	If M0A0 scored (i.e. no algebraic elimination seen), **SC B1** can be awarded for both answers correct Consistent use of σ^2 or $\sqrt{\sigma}$ throughout apply **MR** penalty to A mark or SC B mark.
		5	

309. 9709_s19_ms_63 Q: 1

Question	Answer	Marks	Guidance
(i)	$P(79 < X < 91) = P\left(\frac{79-85}{6.8} < Z < \frac{91-85}{6.8}\right)$ $= P(-0.8824 < Z < 0.8824)$	M1	Using ± standardisation formula for either 79 or 91, no continuity correction
	$= \Phi(0.8824) - \Phi(-0.8824)$ $= 0.8111 - (1 - 0.8111)$	M1	Correct area ($\Phi - \Phi$) with one +ve and one –ve z-value or $2\Phi - 1$ or $2(\Phi - 0.5)$
	$= 0.622$	A1	Correct answer
		3	
(ii)	$z = -1.751$	B1	± 1.751 seen
	$-1.751 = \frac{t-85}{6.8}$	M1	An equation using ± standardisation formula with a z-value, condone σ^2 or $\sqrt{\sigma}$
	$t = 73.1$	A1	Correct answer
		3	

310. 9709_s19_ms_63 Q: 5

Question	Answer	Marks	Guidance
(i)	$P(0, 1, 2) = (0.66)^{14} + {}^{14}C_1(0.34)(0.66)^{13} + {}^{14}C_2(0.34)^2(0.66)^{12}$	M1	Binomial term of form ${}^{14}C_x p^x (1-p)^{14-x}$ $0 < p < 1$ any p, $x \neq 14,0$
	$= 0.0029758 + 0.02146239 + 0.071866$	A1	Correct unsimplified answer
	$= 0.0963$	A1	Correct answer
		3	
(ii)	Mean $=600 \times 0.34 = 204$, Var $= 600 \times 0.34 \times 0.66 = 134.64$	B1	Correct unsimplified np and npq (or sd = 11.603 or Variance = 3366/25)
	$P(< 190) = P\left(z < \frac{189.5-204}{\sqrt{134.64}}\right) = P(z < -1.2496)$	M1	Substituting *their* μ and σ, (no σ^2 or $\sqrt{\sigma}$) into the Standardisation Formula with a numerical value for *'189.5'*. Condone ± standardisation formula
		M1	Using continuity correction 189.5 or 190.5 within a Standardisation formula
	$= 1 - \Phi(1.2496)$	M1	Appropriate area Φ from standardisation formula P(z<….) in final solution, (<0.5 if z is –ve, >0.5 if z is +ve)
	$= 1 - 0.8944 = 0.106$	A1	Correct final answer
		5	

311. 9709_w19_ms_61 Q: 7

Question	Answer	Marks	Guidance
(i)	$P(46 < X < 53) = P\left(\frac{46-49.2}{2.8} < Z < \frac{53-49.2}{2.8}\right)$	M1	Using ± standardisation formula for either 46 or 53, no continuity correction, σ^2 or $\sqrt{\sigma}$
	$P(-1.143 < Z < 1.357)$	A1	Both standardisations correct unsimplified
	$\Phi(1.357) + \Phi(1.143) - 1$ $= 0.9126 + 0.8735 - 1$	M1	Correct final area
	0.786	A1	Final answer
		4	

Question	Answer	Marks	Guidance
(ii)	$\frac{t-49.2}{2.8}=-1.406$	**B1**	±1.406 seen
		M1	An equation using ± standardisation formula with a z-value, condone σ^2 or $\sqrt{\sigma}$
	45.3	**A1**	
		3	
(iii)	$P(X<46)=0.1265$	**M1**	Calculated or ft from **(i)**
	$P(2PB<46)=3(1-0.1265)0.1265^2$	**M1**	$3(1-p)p^2$, $0<p<1$
	0.0419	**A1**	
		3	

312. 9709_w19_ms_62 Q: 4

Question	Answer	Marks	Guidance
(i)	$P(8, 9, 10) = {}^{10}C_8\ 0.66^8\ 0.34^2 + {}^{10}C_9\ 0.66^9\ 0.34^1 + 0.66^{10}$	**M1**	Correct binomial term, ${}^{10}C_a\ 0.66^a(1-0.66)^b$ $a+b=10$, $0<a,b<10$
		A1	Correct unsimplified expression
	0.284	**B1**	CAO
		3	

Question	Answer	Marks	Guidance
(ii)	$np=0.66\times150=99$ $npq=0.66\times(1-0.66)\times150=33.66$	**B1**	Accept evaluated or unsimplified μ, σ^2 numerical expressions, condone $\sigma=\sqrt{33.66}=5.8017$ or 5.802 CAO
	$P(X>84)=P\left(Z>\frac{84.5-99}{\sqrt{33.66}}\right)$	**M1**	± Standardise, $\frac{x-their\,99}{\sqrt{their\,33.66}}$, condone σ^2, x a value
		M1	84.5 or 83.5 used in *their* standardisation formula
	$(=P(Z>-2.499))$	**M1**	Correct final area
	0.994	**A1**	Final answer (accept 0.9938) SC if no standardisation formula seen, B2 $P(Z>-2.499)=0.994$
		5	

313. 9709_w19_ms_62 Q: 6

Question	Answer	Marks	Guidance
(i)	$P(X<45)=P\left(Z<\frac{45-40}{8}\right)$ $=P(Z<0.625)$	M1	± Standardise, no continuity correction, σ^2 or $\sqrt{\sigma}$, formula must be seen
	0.734(0)	A1	CAO
		2	
(ii)	$1-2(1-(i))=2(i)-1=2((i)-0.5)$	M1	Use result of **part (i)** or recalculated to find area OE
	0.468	A1ft	0 < FT from **(i)** < 1 or correct.
		2	
(iii)	$P(X<10)=48/500=0.096$ $z=-1.305$	B1	$z=\pm 1.305$
	$P(X>24)=76/500=0.152$ $z=1.028$	B1	$z=\pm 1.028$
	$10-\mu=-1.305\sigma$ $24-\mu=1.028\sigma$	M1	Form 1 equation using 10 or 24 with μ, σ, z–value. Allow continuity correction, not $\sigma^2, \sqrt{\sigma}$
	$14=2.333\sigma$	M1	OE Solve two equations in σ and μ to form equation in one variable
	$\sigma=6.[00], \quad \mu=17.8[3]$	A1	CAO, WWW
		5	

314. 9709_w19_ms_63 Q: 4

Question	Answer	Marks	Guidance
(i)	$P(h<148)=0.67$	B1	$z=\pm 0.44$ seen
	$\frac{h-148}{8}=0.44$	M1	z-value $=\pm\frac{(h-148)}{8}$
	$151.52\approx 152$	A1	CAO
		3	
(ii)	$P(144<X<152)=P\left(\frac{144-148}{8}<Z<\frac{152-148}{8}\right)$	M1	Using ± standardisation formula for either 144 or 152, $\mu=148$, $\sigma=8$ and no continuity correction, allow σ^2 or $\sqrt{\sigma}$
	$=P\left(-\frac{1}{2}<Z<\frac{1}{2}\right)=0.6915-(1-0.6915)=2\times 0.6915-1$	M1	Correct final area legitimately obtained from phi(*their* z_2) – phi(*their* z_1)
	$=0.383$	A1	Final probability answer
	$0.383\times 120=45.96$ Accept 45 or 46 only	B1FT	*Their* prob (to 3 or 4 sf) × 120, rounded to a whole number or truncated
		4	

315. 9709_w19_ms_63 Q: 7

Question	Answer	Marks	Guidance
(i)(a)	$P(0, 1, 2) = {}^6C_0\ 0.3^0\ 0.7^6 + {}^6C_1\ 0.3^1\ 0.7^5 + {}^6C_2\ 0.3^2\ 0.7^4$	M1	Binomial term of form ${}^6C_x p^x(1-p)^{6-x}$ $0<p<1$ any p, $x\neq 6,0$
	0.1176 ... + 0.3025 ... + 0.3241 ...	A1	Correct unsimplified answer
	0.744	A1	Correct final answer
		3	

Question	Answer	Marks	Guidance
(i)(b)	P(support neither choir) = 1 – (0.3 + 0.45) = 0.25	**M1**	0.25^n seen alone, $1 < n \leqslant 6$
	P(6 support neither choir) = 0.25^6 = 0.000244 or $\frac{1}{4096}$	**A1**	Correct final answer
		2	
(ii)	Mean = 240 × 0.25 = 60 Variance = 240 × 0.25 × 0.75 = 45	**B1FT**	Correct unsimplified *240p* and *240pq* where p =*their* P(support neither choir) or 0.25
	$P(X < 50) = P\left(Z < \frac{49.5-60}{\sqrt{45}}\right) = P(Z < -1.565)$	**M1**	Substituting *their* μ and σ (condone σ^2) into the ±Standardisation Formula with a numerical value for *'49.5'*.
		M1	Using continuity correction 49.5 or 50.5 within a standardisation expression
	1 – 0.9412	**M1**	Appropriate area Φ from standardisation formula P(z< …) in final solution, (< 0.5 if z is –ve, > 0.5 if z is +ve)
	0.0588	**A1**	Correct final answer
		5	

316. 9709_m18_ms_62 Q: 7

Question	Answer	Marks	Guidance
(i)	$P(X > 410) = 225/6000 = 0.0375$ $P\left(Z > \frac{410-400}{\sigma}\right) = 0.0375: 0.9625$	**M1**	Use 1 −225/6000 = 0.9625 to find z value
	z value = ±1.78	**A1**	z value: ± 1.78
	$\frac{10}{\sigma} = 1.78$	**M1**	(410-400)/σ = *their* z (must be a z value)
	$\sigma = 5.62$	**A1**	
		4	
(ii)	We need $P(Z < -1.5)$ and $P(Z > 1.5)$	**M1**	Attempt at $P(Z < -1.5)$ or $P(Z > 1.5)$ 1 – Φ(1.5) seen
	$\Phi(-1.5) + 1 - \Phi(1.5)$ $= 2 - 2\Phi(1.5)$	**M1**	Or equivalent expression with values
	=2 – 2 × 0.9332 = 0.1336 (0.134)	**A1**	Correct to 3sf
	Number expected = 500 × 0.1336 = 66.8: 66 or 67 packets	**B1ft**	0.1336 used or FT their 4sf probability times 500, (not 0.9625 or 0.0375) rounded or truncated
		4	

317. 9709_m18_ms_62 Q: 8

Question	Answer	Marks	Guidance
(i)	$P(4) + P(5) = {}^5C_4\left(\frac{1}{4}\right)^4\left(\frac{3}{4}\right)^1 + {}^5C_5\left(\frac{1}{4}\right)^5\left(\frac{3}{4}\right)^0$	**M1**	One binomial term, with $p < 1$, $n=5$, $p + q=1$
	= 0.014648.. + 0.00097656..	**M1**	Add 2 correct unsimplified binomial terms
	= 0.0156 or $\frac{1}{64}$	**A1**	
		3	
(ii)	$1 - P(0) > 0.995$: $0.75^n < 0.005$	**M1**	Equation or inequality involving 0.75^n and 0.005 or 0.25^n and 0.995
	$n\log 0.75 < \log 0.005$ $n > 18.4$:	**M1**	Attempt to solve *their* exponential equation using logs, or trial and error May be implied by their answer
	$n = 19$	**A1**	
		3	
(iii)	$p = 0.25$, $n = 160$: mean = 160 x 0.25 (= 40) variance = 160 x 0.25 x 0.75 (=30)	**B1**	Correct unsimplified mean and variance
	$P(X < 50) = P\left(Z < \frac{49.5 - 40}{\sqrt{30}}\right)$	**M1**	Use standardisation formulae must include square root.
		M1	Use continuity correction ±0.5 (49.5 or 50.5)
	= P(Z < 1.734) = 0.959	**A1**	Correct final answer
		4	

318. 9709_s18_ms_61 Q: 4

Question	Answer	Marks	Guidance
(a)	$z_1 = 2.4$	**B1**	± 2.4 seen accept 2.396
	$z_2 = -0.5$	**B1**	± 0.5 seen
	$2.4 = \frac{36800 - \mu}{\sigma}$	**M1**	Either standardisation eqn with z value, not 0.5082, 0.7565, 0.0082, 0.6915, 0.3085, 0.6209, 0.0032 or any other probability
	$-0.5 = \frac{31000 - \mu}{\sigma}$	**M1**	Sensible attempt to eliminate μ or σ by substitution or subtraction from their 2 equations (z-value not required), need at least 1 value stated
	σ = 2000 μ = 32000	**A1**	Both correct answers
		5	
(b)	$P(X < 3\mu) = P\left(z < \frac{3\mu - \mu}{(4\mu/3)}\right)$ or $P = \left(z < \frac{(9\sigma/4) - (3\sigma/4)}{\sigma}\right)$	**M1**	Standardise, in terms of one variable, accept σ^2 or $\sqrt{\sigma}$
	$P\left(z < \frac{6}{4}\right)$	**M1**	$\frac{6}{4}$ or $\frac{6}{4\sigma}$ seen
	= 0.933	**A1**	Correct final answer
		3	

319. 9709_s18_ms_61 Q: 5

Question	Answer	Marks	Guidance
(i)	$P(4, 5, 6) = {}^{15}C_4(0.22)^4(0.78)^{11} + {}^{15}C_5(0.22)^5(0.78)^{10} +$	M1	One binomial term ${}^{15}C_x p^x(1-p)^{15-x}$ $0 < p < 1$
	${}^{15}C_6(0.22)^6(0.78)^9$	A1	Correct unsimplified expression
	$= 0.398$	A1	Correct answer
		3	
(ii)	$\mu = 145 \times 0.22 = 31.9$ $\sigma^2 = 145 \times 0.22 \times 0.78 = 24.882$	B1	Correct unsimplified mean and variance
	$P(x > 26) = P\left(z > \frac{26.5 - 31.9}{\sqrt{24.882}}\right) = P(z > -1.08255)$	M1	Standardising must have sq rt
		M1	25.5 or 26.5 seen as a cc
	$= \Phi(1.08255)$	M1	Correct area Φ, must agree with their μ
	$= 0.861$	A1	Correct final answer accept 0.861, or 0.860 from 0.8604 not from 0.8599
		5	

320. 9709_s18_ms_62 Q: 3

Question	Answer	Marks	Guidance
(i)	$z = -1.282$	B1	± 1.282 seen
	$-1.282 = \frac{440 - \mu}{9}$	M1	±Standardisation equation with 440, 9 and μ, equated to a z-value, (not $1 - z$-value or probability e.g. 0.1841, 0.5398, 0.6202, 0.8159)
	$\mu = 452$	A1	Correct answer rounding to 452, not dependent on B1
		3	
(ii)	$P(z > 1.8) = 1 - 0.9641 = 0.0359$	B1	
	Number $= 0.0359 \times 150$ $= 5.385$	M1	$p \times 150$, $0 < p < 1$
	(Number of cartons =) 5	A1FT	Accept either 5 or 6, not indicated as an approximation, e.g. ~, about **FT** *their* $p \times 150$, answer as an integer
		3	

321. 9709_s18_ms_62 Q: 7

Question	Answer	Marks	Guidance
(i)	**Method 1** P(< 11) = 1 – P(11, 12, 13)	**M1**	Binomial expression of form $^{13}C_x\ (p)^x(1-p)^{13-x}$, $0 < x < 13$, $0 < p < 1$
	$= 1 - {}^{13}C_{11}(0.6)^{11}(0.4)^2 - {}^{13}C_{12}(0.6)^{12}(0.4) - (0.6)^{13}$	**M1**	Correct unsimplified answer
	= 0.942	**A1**	CAO
	Method 2 P(< 11) = P(0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10)	**M1**	Binomial expression of form $^{13}C_x\ (p)^x(1-p)^{13-x}$ $0 < x < 13$, $0 < p < 1$
	$= (0.4)^{13} + {}^{13}C_1(0.4)^{12}(0.6) + \ldots + {}^{13}C_{10}(0.4)^3(0.6)^{10}$	**M1**	Correct unsimplified answer
	= 0.942	**A1**	CAO
		3	
(ii)	$\mu = 130 \times 0.35 = 45.5$ var = $130 \times 0.35 \times 0.65 = 29.575$	**B1**	Correct unsimplified mean and var (condone $\sigma^2 = 29.6$, $\sigma = 5.438$)
	$P(\geqslant 50) = P\left(z > \frac{49.5 - 45.5}{\sqrt{29.575}}\right) = P(z > 0.7355)$	**M1**	Standardising, using $\pm\left(\frac{x - \text{their mean}}{\text{their } \sigma}\right)$, x = value to standardise 49.5 or 50.5 seen in ± standardisation equation
	$= 1 - \Phi(0.7355)$	**M1**	Correct final area
	= 1 – 0.7691	**M1**	
	= 0.231	**A1**	Correct final answer
		5	

Question	Answer	Marks	Guidance
(iii)	$1 - (0.65)^n > 0.98$ or $0.02 > (0.65)^n$	**M1**	Eqn or inequality involving, 0.65^n and 0.02 **or** 0.35^n and 0.98
	$n > 9.08$	**M1**	Attempt to solve their eqn or inequality by logs **or** trial and error
	$n = 10$	**A1**	CAO
		3	

322. 9709_s18_ms_63 Q: 6

Question	Answer	Marks	Guidance
(i)	$z_1 = \pm\frac{4.1 - 5.7}{0.8} = -2$ $\quad z_2 = \pm\frac{5 - 5.7}{0.8} = -0.875$	**M1**	At least one standardising no cc no sq rt no sq using 5.7 and 0.8 and either 4.1 or 5
	P(Toffee Apple) $= P(d < 5.0) - P(d < 4.1)$ $= P(z < -0.875) - P(z < -2)$ $= \Phi(-0.875) - \Phi(-2)$ $= \Phi(2) - \Phi(0.875)$	**M1**	Correct area Φ – Φ legitimately obtained – need 2 negative z-values or 2 positives – not one of each
	= 0.9772 – 0.8092 = 0.168 (or 0.1908 – 0.0228)	**A1**	Correct final answer
	Total:	**3**	
(ii)	np = 250 × 0.168 = 42, $\quad$ npq = 34.944	**B1ft**	Correct unsimplified mean and var – ft their prob for **(i)** providing $(0 < p < 1)$ Implied by $\sigma = \sqrt{34.944} = 5.911$
	$P(< 50) = P\left(z < \frac{49.5 - 42}{\sqrt{34.944}}\right) = P(z < 1.2687)$	**M1**	± Standardising using 50, their mean and sd; must have sq rt.
		M1	49.5 or 50.5 seen as a cc
	$= \Phi(1.2687)$	**M1**	Correct area Φ(> 0.5 for + z and < 0.5 for –z)in their final answer
	= 0.898	**A1**	Correct final answer
	Total:	**5**	

323. 9709_w18_ms_61 Q: 4

Question	Answer	Marks	Guidance
(a)	$P(X < 29.4) = P\left(Z < \frac{29.4 - 31.4}{\sqrt{3.6}}\right)$ $= P(Z < -1.0541)$	**M1**	Standardise, no cc, must have sq rt.
	$= 1 - 0.8540$	**M1**	Obtain 1 − prob
	$= 0.146$	**A1**	Correct final answer
		3	

Question	Answer	Marks	Guidance
(b)	$P(X < 12) = \frac{42}{400} = 0.105$ and $P(X > 19) = \frac{58}{400} = 0.145$	**M1**	Eqn with μ, σ and a z-value. Allow cc, wrong sign, but not $\sqrt{\sigma}$ or σ^2
	$\frac{12 - \mu}{\sigma} = -1.253$	**B1**	Any form with z value rounding to ±1.25
	$\frac{19 - \mu}{\sigma} = 1.058$	**B1**	Any form with z value rounding to ±1.06
	$12 - \mu = -1.253\sigma$ $19 - \mu = 1.058\sigma$ $7 = 2.307\sigma$ or $36.455 + 2.307\mu = 0$ oe	**M1**	Solve 2 equations in μ, σ eliminating to 1 unknown
	$\mu = 15.8$, $\sigma = 3.03$	**A1**	Correct answers
		5	

324. 9709_w18_ms_61 Q: 5

Question	Answer	Marks	Guidance
(i)	$1 - (P(7) + P(8) + P(9))$ $= 1 - ({}^9C_7\ 0.8^7 \times 0.2^2 + {}^9C_8\ 0.8^8 \times 0.2^1 + {}^9C_9\ 0.8^9 \times 0.2^0)$	**M1**	Any binomial term of form ${}^9C_x p^x (1-p)^{9-x}$, $x \neq 0$
		M1	Correct unsimplified expression
	$= 1 - (0.3019899 + 0.3019899 + 0.1342177)$ $= 0.262$	**A1**	Correct answer
		3	

Question	Answer	Marks	Guidance
(ii)	Mean = 200 × 0.8 = 160: var = 200 × 0.8 × 0.2 = 32	**B1**	Both unsimplified
	$P(X > 166) = P\left(Z > \frac{166.5 - 160}{\sqrt{32}}\right)$	**M1**	Standardise, $z = \pm \frac{x - their\,160}{\sqrt{their\,32}}$ with square root
		M1	166.5 or 165.5 seen in attempted standardisation expression
	$= P(Z > 1.149) = 1 - 0.8747$	**M1**	1 – a Φ -value, correct area expression, linked to final answer
	$= 0.125$	**A1**	Correct final answer
		5	
(iii)	$np = 160$, $nq = 40$: both > 5 (so normal approx. holds)	**B1**	Both parts required
		1	

325. 9709_w18_ms_62 Q: 7

Question	Answer	Marks	Guidance
(a)(i)	$P(X<4)=P\left(Z<\frac{4-3.24}{0.96}\right)$	M1	±Standardisation formula, no cc, no sq rt, no square
	$=P(Z<0.7917)=0.7858$	A1	$0.7855<p\leqslant 0.7858$ or p = 0.786 Cao (implies M1A1 awarded), may be seen used in calculation
	their 0.7858 × 365 = 286 (or 287)	B1ft	*Their* probability × 365 provided 4sf probability **seen**. FT answer rounded or truncated to nearest integer. No approximation notation used.
		3	
(a)(ii)	$P(X<k)=P(Z<\frac{k-3.24}{0.96})=0.8$	B1	(z=) ± 0.842 seen
	$\frac{k-3.24}{0.96}=0.842$	M1	$z=\pm\frac{k-3.24}{0.96}$, allow cc, sq rt or square equated to a *z*-value (0.7881, 0.2119, 0.158, 0.8, 0.2 etc. are not acceptable)
	$k=4.05$	A1	Correct final answer, www
		3	
(a)(iii)	$P(-1.5<Z<1.5)=$	M1	$\Phi(z=1.5)$ or $\Phi(z=-1.5)$ seen used or $p=0.9332$ seen
	$\Phi(1.5)-\Phi(-1.5)=2\Phi(1.5)-1$ $=2\times 0.9332-1$ oe	M1	Correct final area expression using *their* probabilities
	$=0.866$	A1	Correct final answer
		3	

Question	Answer	Marks	Guidance
(b)	$P(Y>0)=P\left(Z>\frac{0-\mu}{\sigma}\right)\equiv P\left(Z>\frac{0-\mu}{3\mu/4}\right)$ or $P\left(Z>\frac{0-\left(\frac{4\sigma}{3}\right)}{\sigma}\right)$	M1	±Standardisation attempt in terms of one variable no sq rt or square, condone ±0.5 as cc
	$=P(Z>-4/3)$	A1	Correct unsimplified standardisation, no variables
	$=0.909$	A1	Correct final answer
		3	

326. 9709_w18_ms_63 Q: 5

Question	Answer	Marks	Guidance
(i)	$z_1=\pm\frac{90-120}{24}=-\frac{5}{4}$, $z_2=\pm\frac{140-120}{24}=\frac{5}{6}$	M1	At least one standardisation, no cc, no sq rt, no sq using 120 and 24 and either 90 or 140
	$=\Phi\left(\frac{20}{24}\right)-\Phi\left(-\frac{30}{24}\right)$	A1	–5/4 and 5/6 unsimplified
	$=\Phi(0.8333)-(1-\Phi(1.25))$ $=0.7975-(1-0.8944)$ or $0.8944-0.2025=0.6919$	M1	Correct area Φ – Φ legitimately obtained and evaluated from phi(their z_2) – phi (their z_1)
	$=0.692$ AG	A1	Correct answer obtained from 0.7975 and 0.1056 oe to 4sf or 0.6919 seen www
		4	

Question	Answer	Marks	Guidance
(ii)	**Method 1**		
	Probability = P(2, 3, 4) $= 0.692^2(1-0.692)^2 \times {}^4C_2 + 0.692^3(1-0.692) \times {}^4C_3 + 0.692^4$	**M1**	Any binomial term of form $4C_x p^x (1-p)^{4-x}$, $x \neq 0$ or 4
		B1	One correct bin term with $n = 4$ and $p = 0.692$,
	= 0.27256 + 0.40825 + 0.22931	**M1**	Correct unsimplified expression using 0.692 or better
	= 0.910	**A1**	Correct answer
	Method 2:		
	1 – P(0, 1) =	**M1**	Any binomial term of form $4C_x p^x (1-p)^{4-x}$, $x \neq 0$ or 4
	$1 - 0.692^0(1-0.692)^4 \times {}^4C_0 - 0.692^1(1-0.692)^3 \times {}^4C_1$	**B1**	One correct bin term with $n = 4$ and $p = 0.692$
	= 1 – 0.00899 – 0.0808757	**M1**	Correct unsimplified expression using 0.692 or better
	= 0.910	**A1**	Correct answer
		4	

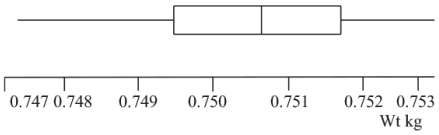
327. 9709_w18_ms_63 Q: 6

Question	Answer	Marks	Guidance
(i)	$P(X>1800) = 0.96$, so $P(Z > \frac{1800-2000}{\sigma}) = 0.96$	**B1**	± 1.75 seen
	$\Phi(\frac{200}{\sigma}) = 0.96$ $\frac{200}{\sigma} = 1.751$	**M1**	$z = \pm\frac{1800-2000}{\sigma}$, allow cc, allow sq rt, allow sq equated to a z-value
	$\sigma = 114$	**A1**	Correct final answer www
		3	
(ii)	Mean = 300 × 0.2 = 60 and variance = 300 × 0.2 × 0.8 = 48	**B1**	Correct unsimplified mean and variance
	$P(X < 70) = P(Z > \frac{69.5-60}{\sqrt{48}})$	**M1**	$Z = \pm\frac{x - their\,60}{\sqrt{their\,48}}$
	$= \Phi(1.371)$	**M1**	69.5 or 70.5 seen in an attempted standardisation expression as cc
	=0.915	**A1**	Correct final answer
		4	
(iii)	np = 60, nq = 240: both > 5, (so normal approximation holds)	**B1**	Both parts evaluated are required
		1	

328. 9709_m17_ms_62 Q: 3

Question	Answer	Marks	Guidance
	$np = 160 \times 0.1$ (16) $npq = 160 \times 0.1 \times 0.9$ (14.4)	**B1**	Correct unsimplified np and npq
	$P(> 17) = P\left(z > \frac{17.5-16}{\sqrt{14.4}}\right) = P(z > 0.3953)$	**M1**	Standardising need √
		M1	16.5 or 17.5 seen in standardised eqn for continuity correction
	= 1 – 0.6536	**M1**	Correct area from their mean (1 – Φ), final solution
	= 0.346	**A1**	
	Total:	**5**	

329. 9709_m17_ms_62 Q: 4

Question	Answer	Marks	Guidance
(i)	LQ = 0.7495 Med = 0.7507 UQ = 0.7517	M1	Attempt to find all 3 quartiles can be implied, Condone LQ=0.7496, Med=0.7506, UQ=0.7515
		B1	Correct median line in box using their scale
		A1	Correct quartiles in box
		B1	Correct end whiskers(not dots or boxes), lines not through box,
		B1	Correct uniform scale from at least 0.7473 to 0.7532, and label (wt) kg oe can be seen in title or scale
	Total:	**5**	

Question	Answer	Marks	Guidance
(ii)	Normal	B1	
	Symmetrical/peaks in middle or tails off quickly	B1	Need symm + another reason
	Total:	**2**	

330. 9709_m17_ms_62 Q: 7

Question	Answer	Marks	Guidance
(a)(i)	$0.674 = \frac{8.8-\mu}{\sigma} \Rightarrow 0.674\sigma = 8.8-\mu$	B1	±0.674 seen
	$-0.935 = \frac{7.7-\mu}{\sigma} \Rightarrow -0.935\sigma = 7.7-\mu$	B1	±0.935 seen (condone ±0.934)
		M1	An eqn with a z-value, μ and σ allow sq rt, sq cc
		M1	sensible attempt to eliminate μ or σ by substitution or subtraction
	$\sigma = 0.684$ $\mu = 8.34$	A1	correct answers (from –0.935)
	Total:	**5**	
(a)(ii)	$P(< 8.2) = P\left(z < \frac{8.2-7.9}{0.44}\right)$	M1	Standardising no cc no sq rt no sq
		M1	Correct area ie Φ, final solution
	$= P(z < 0.6818) = 0.7524$	A1	Correct prob rounding to 0.752
	$P(3) = {}^5C_3 (0.7524)^3(0.2476)^2$	M1	Binomial ${}^5C_x$ powers summing to 5, any p, $\Sigma p = 1$
	$= 0.261$	A1	
	Total:	**5**	

Question	Answer	Marks	Guidance
(b)	$P(< 1.5\mu) = P\left(z < \frac{1.5\mu-\mu}{\mu}\right) = P(z < 0.5)$	*M1	standardising with μ and σ (σ may be replaced by μ)
		DM1	just one variable
	$= 0.692$	A1	
	Total:	**3**	

331. 9709_s17_ms_61 Q: 6

Question	Answer	Marks	Guidance
(a)(i)	$z = 0.674$	B1	rounding to ±0.674 or 0.675
	$0.674 = \frac{6.8-\mu}{0.25\mu}$	M1	standardising, no cc, no sq rt, no sq, σ may still be present on RHS
		M1	subst and sensible solving for μ must collect terms, no z-value needed can be 0.75 or 0.7734 need a value for μ
	$\mu = 5.82$	A1	
	Total:	4	
(a)(ii)	$P(X < 4.7) = P\left(z < \frac{4.7-5.819}{1.4548}\right)$	M1	± standardising no cc, no sq rt, no sq unless penalised in (a)(i)
	= ϕ(–0.769) = 1 – 0.7791	M1	correct side for their mean i.e. 1–ϕ (final solution)
	= 0.221	A1	
	Total:	3	
(b)	$P(<15.75) = P\left(z < \frac{15.75-16}{0.2}\right) = 1 - P(z < 1.25) = 1 - 0.8944 = 0.1056$ and P(>16.25) = 0.1056 by sym	*M1	Standardising for 15.75 or 16.25 no cc no sq no sq rt unless penalised in (a)(i) or (a)(ii)
	P(usable) = 1 – 0.2112 = 0.7888	B1	2ϕ– 1 OE for required prob, (final solution)
	Usable rods=1000 × 0.7888 =	DM1	Mult their prob by 1000 dep on recognisable attempt to standardise
	788 or 789	A1	
	Total:	4	

332. 9709_s17_ms_62 Q: 5

(i)	$(z =) \frac{4.2-3.9}{\sigma}$	M1	Standardising, not square root of σ, not σ^2
	$z = 0.916$ or 0.915	B1	Accept $0.915 \leqslant \pm z \leqslant 0.916$ seen
	$\sigma = 0.328$	A1	Correct final answer (allow 20/61 or 75/229)
	Total:	3	

Question	Answer	Marks	Guidance
(ii)	$z = 4.4 - 3.9$/their 0.328 or $z = 3.4 - 3.9$/their 0.328 = 1.5267 = –1.5267	M1	Standardising attempt with 3.4 or 4.4 only, allow square root of σ, or σ^2
	$\Phi = 0.9364$	A1	$0.936 \leqslant \Phi \leqslant 0.937$ or $0.063 \leqslant \Phi \leqslant 0.064$ seen
	Prob = 2Φ – 1 = 2(0.9364) – 1	M1	Correct area 2Φ – 1OE i.e. Φ = – (1 – Φ), linked to final solution
	= 0.873	A1	Correct final answer from $0.9363 \leqslant \Phi \leqslant 0.9365$
	Total:	4	
(iii)	dividing (0.5) by a larger number gives a smaller z-value or more spread out as sd larger or use of diagrams	*B1	No calculations or calculated values present e.g. ($\sigma =$)0.656 seen Reference to spread or z value required
	Prob is less than that in (ii)	DB1	Dependent upon first B1
	Total:	2	

333. 9709_s17_ms_63 Q: 2

	$np = 270 \times 1/3 = 90$, $npq = 270 \times 1/3 \times 2/3 = 60$	**B1**	Correct unsimplified np and npq, SOI
	$P(x > 100) = P\left(z > \frac{99.5 - 90}{\sqrt{60}}\right) = P(z > 1.2264)$	**M1** **M1**	±Standardising using 100 need sq rt Continuity correction, 99.5 or 100.5 used
	$= 1 - 0.8899$	**M1**	Correct area $1 - \Phi$ implied by final prob. < 0.5
	$= 0.110$	**A1**	
	Total:	**5**	

334. 9709_s17_ms_63 Q: 4

(a)	$P(x > 0) = P\left(z > \pm\frac{0 - \mu}{\sigma}\right)$ $= P\left(z > \frac{-\mu}{\mu/1.5}\right)$ or $P\left(z > \frac{-1.5\sigma}{\sigma}\right)$	**M1**	±Standardising, in terms of μ and/or σ with 0 - …. in numerator, no continuity correction, no √
	$= P(z > -1.5)$	**A1**	Obtaining z value of ±1.5 by eliminating μ and σ, SOI
	$= 0.933$	**A1**	
	Total:	**3**	
(b)	$z = -1.151$	**B1**	± z value rounding to 1.1 or 1.2
	$-1.151 = \frac{70 - 120}{s}$	**M1**	± Standardising (using 70) equated to a z-value, no cc, no squaring, no √
	$\sigma = 43.4$ or 43.5	**A1**	
	Totals:	**3**	

335. 9709_w17_ms_61 Q: 7

Question	Answer	Marks	Guidance
(i)	$P(< 570) = P\left(z < \frac{570 - 500}{91.5}\right) = P(z < 0.7650)$ $= 0.7779$	**M1**	Standardising for either 570 or 390, no cc, no sq, no √
	$P(< 390) = P\left(z < \frac{390 - 500}{91.5}\right) = P(z < -1.202)$	**A1**	One correct z value
	$= 1 - 0.8853 = 0.1147$	**A1**	One correct Φ, final solution
	Large:0.222 (0.2221) Small: 0.115 (0.1147)	**A1**	Correct small and large
	Medium: 0.663 (0.6632)	**A1FT**	Correct Medium rounding to 0.66 or ft 1 – (their small + their large)
		5	

Question	Answer	Marks	Guidance
(ii)	$1.645 = \left(\frac{x-500}{91.5}\right)$	B1	± 1.645 seen (critical value)
		M1	Standardising accept cc, sq, sq rt
	$x = 651$	A1	650 ⩽ Ans ⩽ 651
		3	
(iii)	P(x > 610) = 0.1147 (symmetry)	M1	Attempt to find upper end prob x > 610 or Φ(x), ft their P(< 390) from **(i)**
	0.3 + 0.1147 = 0.4147 ⇒ Φ(x) = 0.5853	M1	Adding 0.3 to *their* P(x > 610) or subt 0.5 from Φ(x) or 0.8853 – 0.3
	z = 0.215 or 0.216	M1	Finding $z = \Phi^{-1}(0.5853)$
	$0.215 = \frac{k-500}{91.5}$	M1	Standardising and solving, accept cc, sq, sq rt
	k = 520	A1	
		5	

336. 9709_w17_ms_62 Q: 5

Question	Answer	Marks	Guidance
(i)	*EITHER:* P(> 2) = 1 – P(0, 1, 2)	(M1	Binomial term of form ${}^{30}C_x p^x (1-p)^{30-x}$, $0 < p < 1$ any p
	$= 1 - (0.96)^{30} - {}^{30}C_1(0.04)(0.96)^{29} - {}^{30}C_2(0.04)^2(0.96)^{28}$ (= 1 – 0.2938… – 0.3673… – 0.2219…)	A1	Correct unsimplified answer
	= 1-0.883103 = 0.117 (0.116896)	A1)	
	OR: P(> 2) = P(3,4,5,6,….30)	(M1	Binomial term of form ${}^{30}C_x p^x (1-p)^{30-x}$, $0 < p < 1$ any p
	$= {}^{30}C_3(0.04)^3(0.96)^{27} + {}^{30}C_4(0.04)^4(0.96)^{26} + \ldots + (0.04)^{30}$	A1	Correct unsimplified answer
	= 0.117	A1)	
		3	

Question	Answer	Marks	Guidance
(ii)	$np = 280 \times 0.1169 = 32.73$, $npq = 280 \times 0.1169 \times 0.8831 = 28.9$	M1 FT	Correct unsimplified np and npq, FT their p *from* **(i)**,
	$P(\geqslant 30) = P\left(z > \frac{29.5 - 32.73}{\sqrt{28.9}}\right) = P(z > -0.6008)$	M1	Substituting *their* μ and σ ($\sqrt{npq}$ only) into the Standardisation Formula
		M1	Using continuity correction of 29.5 or 30.5
		M1	Appropriate area Φ from standardisation formula P(z >….) in final solution
	= 0.726	A1	
		5	

337. 9709_w17_ms_62 Q: 7

Question	Answer	Marks	Guidance
(i)	$P(> 65) = P\left(z > \frac{65 - 61.4}{12.3}\right) = P(z > 0.2927)$	M1	Standardising no continuity correction, no square or square root, condone ± standardisation formula
		M1	Correct area (< 0.5)
	= 1 – 0.6153 = 0.385	A1	
		3	

Question	Answer	Marks	Guidance
(ii)	P (< 65) = 0.6153 so P(< k) = 0.25 + 0.6153 = 0.8653	**B1**	
	z = 1.105	**B1**	z = ± 1.105 seen or rounding to 1.1
	$1.105 = \frac{k - 61.4}{12.3}$	**M1**	standardising allow ±, cc, sq rt, sq. Need to see use of tables backwards so must be a z-value, not 1 – z value.
	k = 75.0	**A1**	Answers which round to 75.0. Condone 75 if supported.
		4	
(iii)	$2.326 = \frac{97.2 - \mu}{\sigma}$	**B1**	± 2.326 seen (Use of critical value)
	$-0.44 = \frac{55.2 - \mu}{\sigma}$	**B1**	± 0.44 seen
		M1	An equation with a z-value, μ, σ and 97.2 or 55.2, allow $\sqrt{\sigma}$ or σ^2
		M1	Algebraic elimination μ or σ from *their* two simultaneous equations
	μ = 61.9 σ = 15.2	**A1**	both correct answers
		5	

338. 9709_w17_ms_63 Q: 7

Question	Answer	Marks	Guidance
(i)	$P(t > 6) = P\left(z > \frac{6 - 5.3}{2.1}\right) = P(z > 0.333)$	**M1**	Standardising, no continuity correction, no sq, no sq rt
	= 1 – 0.6304	**M1**	Correct area 1 – Φ (< 0.5), final solution
	= 0.370 or 0.369	**A1**	
		3	
(ii)	z = 1.645	**B1**	± 1.645
	$1.645 = \frac{x - 5.3}{2.1}$	**M1**	Standardising, no continuity correction, allow sq, sq rt. Must be equated to a z-value
	x = 8.75 or 8.755 or 8.7545	**A1**	
		3	
(iii)	$n = 10, p = 0.05$	**M1**	Bin term ${}^{10}C_x\, p^x (1-p)^{10-x}$
	$P(0, 1, 2) = (0.95)^{10} + {}^{10}C_1(0.05)(0.95)^9 + {}^{10}C_2(0.05)^2(0.95)^8$	**M1**	Correct unsimplified answer
	= 0.988 (0.9885 to 4 sf)	**A1**	
		3	
(iv)	P(misses bus) = P(t < 0)	***M1**	Seeing t linked to zero
	$= P\left(z < \frac{0 - 5.3}{2.1}\right) = P(z < -2.524) = 1 - \Phi(2.524)$ = 1 – 0.9942	**DM1**	Standardising with t = 0, no continuity correction, no sq, no sq rt
	= 0.0058	**A1**	
		3	

339. 9709_m16_ms_62 Q: 7

(i)	$z = -1.645$ $-1.645 = \frac{0.9 - m}{0.35}$ $m = 1.48$	**B1** **M1** **A1**	3	± 1.64 to 1.65 seen Standardising with a z-value accept $(0.35)^2$ Correct answer
(ii)	$P(< 2) = P\left(z < \frac{2 - 1.476}{0.35}\right)$ $= P(z < 1.50)$ $= 0.933$ Prob $= (0.9332)^4$ $= 0.758$	**M1** **M1** **A1** **M1** **A1**	5	Standardising no sq , FT *their m*, no cc Correct area i.e. F Accept correct to 2sf here Power of 4, from attempt at P(z) Correct answer

(iii)	$P(t > 0.6\mu) = P\left(z > \frac{0.6\mu - \mu}{\mu / 3}\right)$ $= P(z > -1.2)$ $= 0.885$	**M1** **M1** **A1**	3	Standardising attempt with 1 or 2 variables Eliminating μ or σ Correct final answer

340. 9709_s16_ms_61 Q: 1

Question	Answer	Marks		Guidance
	$z = 1.037$ $1.037 = \frac{1.8 - 1.62}{\sigma}$ $\sigma = 0.18/1.037 = 0.174$	**B1** **M1** **A1**	[3]	Rounding to 1.04 Standardising attempt allow cc no sq rt must have a z-value i.e. not 0.8023 or 0.5596.

341. 9709_s16_ms_61 Q: 5

(i)	$P(x < 3.0) = P\left(z < \frac{3.0 - 2.6}{0.25}\right)$ $+ P(z < 1.6) = 0.945$	**M1** **M1** **A1**	[3]	Standardising no sq rt no cc Correct area i.e. prob > 0.5 legit
(ii)	$X \sim B(500, 0.9452) \sim N(472.6, 25.898)$ $P\left(z > \frac{479.5 - 472.6}{\sqrt{25.89848}}\right) = P(z > 1.3558)$ $= 1 - 0.9125 = 0.0875$	**M1** **M1** **M1** **M1** **A1**	[5]	500 ×'0.9452' and 500×'0.9452'×('1 – 0.9452') seen oe Standardising must have sq rt. All M marks indep cc either 479.5 or 480.5 seen correct area i.e. < 0.5

Question	Answer	Marks	Guidance
(iii)	500× 0.9452 and 500× (1–0.9452) are both > 5	**B1**√ [1]	must see at least 500× 0.0548 > 5oe ft their (i) accept $np > 5$, $nq > 5$ if both not $npq > 5$

342. 9709_s16_ms_62 Q: 2

(i)	0.72	**B1** [1]	
(ii)	$np = 180\times0.72$, $npq = 180\times0.72\times0.28$ X~N(129.6, 36.288)	**B1**√	180 × 0.72, 180 × 0.72 × 0.28 seen, their values or correct
	$P(x>115) = P\left(z > \frac{115.5-129.6}{\sqrt{36.288}}\right)$	**M1**	Standardising (±) must have sq rt
		M1	cc either 115.5 or 114.5 seen
	$= P(z > -2.341)$	**M1**	Correct area, Φ from final answer attempt fully correct method
	$= 0.990$	**A1** [5]	

343. 9709_s16_ms_62 Q: 6

(i)	$P(x > 10.2) = P\left(z > \frac{10.2-9.5}{1.3}\right)$	**M1**	Standardising allow cc, sq rt, sq
	$= P(z > 0.53846)$ $= 1 - 0.7046$	**M1**	1 – Φ final solution attempt
	$= 0.295$	**A1** [3]	
(ii)	$z = -1.282$	**B1**	± rounding to 1.28 seen
	$-1.282 = \frac{t-9.5}{1.3}$	**M1**	Standardising correctly can be ± z value here
	$t = 7.83$	**A1** [3]	Correct answer from $z = -1.282$ only
(iii)	$P(x < 8.8) = 0.2954$ by symmetry	**B1**	oe method, FT *their 0.2954 from (i)*
	Days = 365×0.2954	**M1**	Mult a probability <1 by 365
	= 107 or 108	**A1** [3]	Correct answer (no decimals)

344. 9709_s16_ms_63 Q: 5

(i)	$z = 1.015$	**B1**	Accept z between ±1.01 and 1.02
	$1.015 = \frac{70-69}{\sigma}$	**M1**	Standardising
	$\sigma = 0.985$ (200/203)	**A1** [3]	
(ii)	58 + 9 = 67	**M1**	58 + 9 seen or implied (or 69-58 or 69-9)
	$P(>67) = P\left(z > \frac{67-69}{0.9852}\right)$	**M1**	Standardising ± z no cc allow their sd (must be +ve) Alt. 1 69-58 =11, $P(>9) = P\left(z > \frac{9-11}{0.9852}\right)$ Alt.2 69-9 =60, $P(>58) = P\left(z > \frac{58-60}{0.9852}\right)$
	$= P(z > -2.03)$ $= 0.9788$	**M1**	Correct prob area
	300 × 0.9788	**M1**	Multiply their prob (from use of tables) by 300
	= 293.6 so 293	**A1** [5]	– accept 293 or 294 from fully correct working

345. 9709_s16_ms_63 Q: 7

(i)	$^{12}C_8\,(0.65)^8(0.35)^4 + {}^{12}C_9\,(0.65)^9(0.35)^3 + {}^{12}C_{10}\,(0.65)^{10}(0.35)^2$	**M1**		Bin term with $^{12}C_r p^r (1-p)^{12-r}$ seen $r \neq 0$ any $p<1$
		M1		Summing 2 or 3 bin probs p = 0.65 or 0.35, n = 12
	= 0.541	**A1**	[3]	
(ii)	$P(\bar{R}\bar{R}\bar{R}R) = 0.35 \times 0.35 \times 0.35 \times 0.65$	**M1**		Mult 4 probs either $(0.35)^3(0.65)$ or $(0.65)^3(0.35)$
	= 0.0279	**A1**	[2]	
(iii)	P(7) = 0.2039 (unsimplified)	**B1**		$^{12}C_7\,(0.65)^7(0.35)^5$
	Mean = 250×'0.2039' (= 50.9798) Var = 250×'0.2039' × '(1 – 0.2039)' (= 40.5851)	**B1**		Correct unsimplified np and npq using 'their 0.2039' but not 0.65 or 0.35
	$P(> 54) = P\left(\frac{54.5 - 50.9798}{\sqrt{40.5851}}\right)$ = P(z > 0.5526) = 1 – Φ(0.5526) = 1 – 0.7098	**M1** **M1** **M1**		Standardising need sq rt – must be from working with 54 cc either 53.5 or 54.5 correct area < 0.5 i.e. 1 – Φ - must be from working with 54
	= 0.290	**A1**	[6]	

346. 9709_w16_ms_61 Q: 4

(i)	$P(< 1) = P\left(z < \frac{1-1.04}{0.017}\right) = P(z < -2.353)$ = 1 – 0.9907 = 0.0093	**M1** **M1** **A1**	[3]	Standardising no cc, no √ or sq 1 – Φ (final process)
(ii)	expected number 1000 ÷ 1.04 = 961 or 962	**B1**	[1]	Or anything in between
(iii)	$z = -1.765$ $-1.765 = \frac{1-\mu}{0.017}$ =1.03	**B1** **M1** **A1**	[3]	± 1.76 to 1.77 Standardising must have a z-value, allow √ or sq
(iv)	expected number = 1000 ÷ 1.03 = 971 or 970	**B1**√	[1]	Or anything in between, ft their (iii)

347. 9709_w16_ms_62 Q: 3

(i)	Bin (7, 0.8) $P(6, 7) = {}^7C_6\,(0.8)^6(0.2)^1 + (0.8)^7$ = 0.577	**M1** **M1** **A1**	[3]	$^7C_n\,p^n(1-p)^{7-n}$ seen Correct unsimplified expression for P(6,7)
(ii)	mean = 100×0.2 = 20 Var = 100×0.2×0.8 = 16 $P(\text{at most } 30) = P\left(z < \frac{30.5 - 20}{\sqrt{16}}\right)$ = P(z < 2.625) = 0.996	**B1** **M1** **M1** **M1** **A1**	[5]	Correct unsimplified mean and var Standardising must have sq rt, their μ, variance cc either 29.5 or 30.5 Correct area Φ , from final process

348. 9709_w16_ms_62 Q: 4

(i)	P(< 4.5) = $P\left(z < \frac{4.5-4.2}{0.6}\right)$ = P(z < 0.5) = 0.6915 P(< 3.5) = $P\left(z < \frac{3.5-4.2}{0.6}\right)$ = P(z< -1.167) = 1 – 0.8784 = 0.1216 0.6915 – 0.1216 = 0.570	**M1** **M1** **A1**	 [3]	Standardising once no cc no sq no sq rt $\Phi_1 - (1 - \Phi_2)$ [$P_1 - P_2$, $1>P_1>0.5$, $0.5>P_2>0$] oe
(ii)	z = 1.175 $1.175 = \frac{t-4.2}{0.6}$ t = 4.91	**B1** **M1** **A1**	 [3]	±1.17 to 1.18 seen Standardising no cc, allow sq, sq rt with z – value (not ±0.8106, 0.5478, 0.4522, 0.1894, 0.175 etc.) Correct answer from z = 1.175 seen (4sf)
(iii)	$(0.88)^n < 0.003$ $n > \lg(0.003)/\lg(0.88)$ $n > 45.4$ n = 46	**M1** **M1** **A1**	 [3]	Inequality or eqn in 0.88, power correctly placed using n or (n±1), 0.003 or (1 – 0.003) oe Attempt to solve by logs or trial and error (may be implied by answer) Correct integer answer

349. 9709_w16_ms_63 Q: 6

(i)	P(small) = $P\left(z < \frac{95-150}{50}\right)$ = P(z < –1.1) = 1 – 0.8643 = 0.136	**M1** **M1** **A1**	 [3]	± standardising using 95, no cc, no sq, no sq rt 1 – Φ (in final answer)
(ii)	z = 1.282 $1.282 = \frac{x-150}{50}$ x = 214 g	**B1** **M1** **A1**	 [3]	± rounding to 1.28 Standardised eqn in their z allow cc
(iii)	P(small) = 0.1357, P(large) = 0.1357 symmetry P(medium) = 1 – 0.1357×2 = 0.7286 **AG**	**B1**	[1]	Correct answer legit obtained
(b)	Expected cost per banana = 0.1357×10 + 0.1357×25 + 0.7286×20 = 19.3215 cents Total cost of 100 bananas = 1930 (cents) ($19.30)	***M1** **DM1** **A1**	 [3]	Attempt at multiplying each 'prob' by a price and summing Mult by 100

350. 9709_w16_ms_63 Q: 7

(i)	$P(2) = {}^7C_2(0.1)^2(0.9)^5$ $= 0.124$	**M1** **A1**	[2]	Bin term ${}^7C_2p^2(1-p)^5 \quad 0 < p < 1$
(ii)	$(0.15)^1(0.1)^2(0.75)^2 \times 5!/2!2!$ $= 0.0253$ or $81/3200$	**M1** **M1** **A1**	 [3]	Mult probs for options, $(0.15)^a(0.1)^b(0.75)^c$ where a + b + c sum to 5 Mult by 5!/2!2! oe
(iii)	mean = 365×0.15 (= 54.75 or 219/4) Var = 365× 0.15×0.85 (= 46.5375 or 3723/80) $P(x > 44) = P\left(z > \frac{44.5 - 54.75}{\sqrt{46.5375}}\right)$ $= P(z > -1.5025)$ $= 0.933$	**B1** **M1** **M1** **M1** **A1**	 [5]	Correct unsimplified mean **and** var, oe ± Standardising need sq rt cc either 44.5 (or 43.5) Φ Correct answer accept 0.934

351. 9709_s15_ms_61 Q: 1

	$P(x < 3.273) = 0.5 - 0.475 = 0.025$	**M1**	Attempt to find z-value using tables in reverse
	$z = -1.96$	**A1**	±1.96 seen
	$\frac{3.2 - \mu}{0.714} = -1.96$	**M1**	Solving their standardised equation z-value not nec
	$\mu = 4.60$s	**A1** [4]	Correct ans accept 4.6

352. 9709_s15_ms_61 Q: 6

(i)	$P(5, 6, 7) = {}^8C_5(0.68)^5(0.32)^3 + {}^8C_6(0.68)^6(0.32)^2 + {}^8C_7(0.68)^7(0.32)$ $= 0.722$	M1 M1 A1 A1 [4]	Binomial term ${}^8C_x\,p^x(1-p)^{8-x}$ seen $0 < p < 1$ Summing 3 binomial terms Correct unsimplified answer Correct answer
(ii)	$np = 340$, $npq = 108.8$ $P(x > 337) = P\left(z > \frac{337.5 - 340}{\sqrt{108.8}}\right)$ $= P(z > -0.2396)$ $= 0.595$	B1 M1 M1 M1 A1 [5]	Correct (unsimplified) mean and var standardising with sq rt must have used 500 cc either 337.5 or 336.5 correct area (> 0.5) must have used 500 correct answer
(iii)	$np\,(340) > 5$ and $nq(160) > 5$	B1 [1]	must have both or at least the smaller, need numerical justification

353. 9709_s15_ms_62 Q: 7

	total = 1210	A1	Correct final answer
(a) (i)	prob = $p\left(z < \frac{30-35.2}{4.7}\right)$ = P(z < –1.106) = 1 – 0.8655 = 0.1345 0.1345×52 = 6.99	M1 M1 A1 A1 **4**	Standardising no sq rt no cc no sq 1 – Φ Correct ans rounding to 0.13 Correct final answer accept 6 or 7 if 6.99 not seen but previous prob 0,1345 correct
(ii)	Φ (t) = 0.648 z = 0.380 $0.380 = \frac{t-35.2}{4.7}$ t = 37.0	B1 M1 A1 **3**	0.648 seen standardising allow cc, sq rt,sq, need use of tables not 0.148, 0.648, 0.352, 0.852 correct answer rounding to 37.0
(b)	$\frac{7-\mu}{\sigma} = -0.8\sigma$ so $7-\mu = -0.8\sigma$ $\frac{10-\mu}{\sigma} = 0.44$ so $10-\mu = 0.44\sigma$ μ = 8.94 σ = 2.42	B1 B1 M1 M1 A1 **5**	± 0.8 seen ± 0.44 seen An eqn with z-value, μ and σ no sq rt no cc no sq Sensible attempt to eliminate μ or σ by subst or subtraction, need at least one value Correct answers

354. 9709_s15_ms_63 Q: 1

	z = 1.136 $1.136 = \frac{195-\mu}{22}$ μ = 170	B1 M1 A1 **[3]**	± 1.136 seen, not ±1.14, Standardising, no cc no sq rt, equated to their z not 0.128 or 0.872 Correct answer, nfww

355. 9709_s15_ms_63 Q: 3

	μ = 300×0.072 = 21.6, σ^2 = 20.0448 $P(x < 18) = P\left(z < \frac{17.5-21.6}{\sqrt{20.0448}}\right)$ =P (z < -0.9157) = 1 – 0.8201 = 0.180	B1 M1 M1 M1 A1 **[5]**	300×0.072 seen and 300×0.072×0.928 seen or implied (σ = 4.4771,σ^2 = 20(.0)) oe ±Standardising, their mean/var, with sq root Cont corr 17.5 or 18.5 Correct area 1 - Φ Answer wrt 0.180, nfww

356. 9709_s15_ms_63 Q: 5

(i)	$P(\text{large}) = 1 - \Phi\left(\frac{29-21.7}{6.5}\right)$ $= 1 - \Phi(1.123) = 1 - 0.8692$ $= 0.1308$ $P(0,1) = (0.8692)^{8} + {}^{8}C_1(0.1308)(0.8692)^{7}$ $= 0.718$	M1 M1 A1 M1 M1 A1 **[6]**	Standardising no cc no sq rt Correct area $1 - \Phi$ Rounding to 0.13 Any bin term with ${}^{8}C_x p^x (1-p)^{8-x}$ $0 < p < 1$ Summing bin P(0) + P(1) only with $n = 8$, oe Correct ans
(ii)	$= 1 - (0.8692)^n > 0.98$ $(0.8692)^n < 0.02$ Least number = 28	M1 M1 A1 **[3]**	eq/ineq involving their $(0.8692)^n$ or $(0.1308)^n$, 0.02 or 0.98 oe with or without a 1 solving attempt (could be trial and error) – may be implied by their answer correct answer

357. 9709_w15_ms_61 Q: 2

	$\mu = 54.1$ $z = -1.11$ $-1.11 = \frac{50.9-54.1}{\sigma}$ $\sigma = 2.88$	**B1** **B1** **M1** **A1** [4]	Stated or evaluated Accept rounding to ± 1.1 Standardising no cc no sq rt Correct answer

358. 9709_w15_ms_61 Q: 4

(i)	$\bar{x} = 80 - 147/30 = 80 - 4.9$ $= 75.1$ $\text{sd} = \sqrt{\left(\frac{952}{30} - \left(\frac{147}{30}\right)^2\right)} = \sqrt{7.72\ldots}$ sd = 2.78	**M1** **A1** **M1** **A1** [4]	For –147/30 oe seen Correct answer 952/30 – (± their coded mean)^2 Correct answer
(ii)	$P(x > 160) = P\left(z > \frac{160-148.6}{18.5}\right)$ $= P(z > 0.616)$ $= 1 - 0.7310$ $= 0.269$	**M1** **M1** **A1** [3]	Standardising no cc no sq rt $1 - \Phi$ Correct answer

359. 9709_w15_ms_61 Q: 7

(i)	let P(2, 4, 6) all = p then P(1, 3, 5) all = 2p $3p + 6p = 1$ $p = 1/9$ so prob (3) = 2/9 (0.222)	**M1** **M1** **A1** [3]	Using P(even) = 2P(odd) or vice versa oe Summing P(odd+ even) or P(1, 2, 3, 4, 5, 6) = 1 Correct answer
(ii)	P(5, 5, 6) = 2/9 × 2/9 × 1/9 × ${}^3C_2$ = 4/243 (0.0165)	**M1** **M1** **A1** [3]	Mult three probs together Mult by 3 oe ie summing 3 options Correct answer
(iii)	$\mu = 100\times1/3 = 33.3$, $\sigma = 100\times1/3\times2/3 = 22.2$ $P(x \leqslant 37) = P\left(z \leqslant \frac{37.5 - \frac{100}{3}}{\sqrt{\frac{200}{9}}}\right) = P(z \leqslant 0.8839)$ = 0.812	**B1** **M1** **M1** **M1** **A1** [5]	Unsimplified 100/3 and 200/9 seen Standardising need sq rt 36.5 or 37.5 seen correct area using their mean Correct answer

360. 9709_w15_ms_62 Q: 7

(a) (i)	$P(x > 3900) = P\left(z > \frac{3900 - 4520}{560}\right)$ $= P(z > -1.107) = \Phi(1.107)$ = 0.8657 Number of days = 365 × 0.0.8657 = 315 or 316 (315.98)	**M1** **M1** **A1** **B1√** 4	Standardising no cc no sq rt no sq Correct area Φ ie > 0.5 Prob rounding to 0.866 Correct answer ft their wrong prob if previous A0, $p < 1$, ft must be accurate to 3sf
(ii)	$z = 1.165$ $1.165 = \frac{8000 - m}{560}$ $m = 7350$ (7347.6)	**B1** **M1** **A1** 3	± 1.165 seen Standardising eqn allow sq, sq rt, cc, must have z-value eg not 0.122, 0.878, 0.549, 0.810. Correct answer rounding to 7350
(iii)	$P(0, 1) = (0.878)^6 + {}^6C_1(0.122)^1(0.878)^5$ = 0.840 accept 0.84 Normal approx. to Binomial. M0, M0, A0	**M1** **M1** **A1** 3	Binomial term ${}^6C_x\, p^x(1-p)^{6-x}$ $0< p< 1$ seen Correct unsimplified expression Correct answer
(b)	$P(< 2\mu) = P\left(z > \frac{2\mu - \mu}{\sigma}\right) = P(z < 1.5)$ = 0.933	**M1** **M1** **A1** 3	Standardising with μ and σ Attempt at one variable and cancel Correct answer

361. 9709_w15_ms_63 Q: 4

(i)	$z = 1.127$ $1.127 = \frac{136 - 125}{\sigma}$ $\sigma = 9.76$	**B1** **M1** **A1** 3	± 1.127 seen accept rounding to ±1.13 Standardising no cc no sq rt, with attempt at z (not ±0.8078, ±0.5517, ±0.13, ±0.87) Correct ans

(ii)	P(131<x<141)= P ($\frac{131-125}{9.76}$<z<$\frac{141-125}{9.76}$)	**M1**		Standardising once with their sd, no √,², allow cc
	= Φ(1.639) – Φ(0.6147) = 0.9493 – 0.7307 = 0.2186	**M1**		Correct area Φ2 – Φ1
		M1		Mult by 170, P<1
	Number = 0.2186 × 170 = 37 or 38 or awrt 37.2	**A1**	4	Correct answer, nfww

362. 9709_w15_ms_63 Q: 7

(i)	P(0, 1, 2) = $(0.92)^{19}+{}^{19}C_1(0.08)(0.92)^{18}+{}^{19}C_2(0.08)^2(0.92)^{17}$	**M1** **M1**		Binomial term ${}^{19}C_x p^x(1-p)^{19-x}$ seen 0<p<1 Correct unsimplified expression
	= 0.809	**A1**	3	Correct answer (no working SC B2)
(ii)	P(at least 1) = 1 – P(0) = 1 – P(0.92)^n > 0.90 0.1 > (0.92)^n n > 27.6	**M1** **M1**		Eqn with their 0.92^n, 0.9 or 0.1, 1 not nec Solving attempt by logs or trial and error, power eqn with one unknown power
	Ans 28	**A1**	3	Correct answer, not approx., ≈, ⩾, >, ⩽, <
(iii)	np = 1800 × 0.08 = 144 npq = 132.48	**B1**		correct unsimplified np and npq seen accept 132.5, 132, 11.5, awrt 11.51
	P(at least 152) = P $\left(z > \left(\frac{151.5-144}{\sqrt{132.48}}\right)\right)$	**M1**		standardising, with √
		M1		cont correction 151.5 or 152.5 seen
	= P(z > 0.6516) = 1 – 0.7429 = 0.257	**M1**		correct area 1 – Φ (probability)
		A1	5	correct answer
(iv)	Use because 1800 ×0.08 (and 1800 × 0.92 are both) > 5	**B1**	1	1800 ×0.08 > 5 is sufficient np>5 is sufficient if clearly evaluated in (iii) If npq>5 stated then award B0